MATH 6337 (C. Heil) HOMEWORK #8 Due date: April 28, 2011

Work the following problems and hand in your solutions. You may work together with other people in the class, but you must each write up your solutions independently. A subset of these will be selected for grading. Write LEGIBLY on the FRONT side of the page only, and STAPLE your pages together.

Throughout this homework, we assume that (X, Σ) is a measurable space.

1. Let ν be a signed measure on X, and choose $E \in \Sigma$. Show that

$$\begin{aligned} \nu^+(E) &= \sup\bigl\{\nu(A) : A \in \Sigma,\, A \subseteq E\bigr\}, \\ \nu^-(E) &= -\inf\bigl\{\nu(A) : A \in \Sigma,\, A \subseteq E\bigr\}, \\ |\nu|(E) &= \sup\biggl\{\sum_{k=1}^{n} |\nu(E_k)| : n \in \mathbb{N},\, E_k \in \Sigma,\, E = \bigcup_{k=1}^{n} E_k \text{ disjointly}\biggr\}. \end{aligned}$$

2. Let $\nu = \nu^+ - \nu^-$ be the Jordan decomposition of a signed measure ν on X and let $X = P \cup N$ be an associated Hahn decomposition. Show that $d\nu^+ = \chi_P\, d\nu$, $d\nu^- = -\chi_N\, d\nu$, and $d|\nu| = (\chi_P - \chi_N)\, d\nu$.

3. Show that if ν is a signed measure on X, then

$$|\nu|(E) = \sup\biggl\{\biggl|\int_E f\, d\nu\biggr| : |f| \le 1\biggr\}.$$