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Metrics, norms, inner products, and operator theory. (English) Zbl 1414.46001

Applied and Numerical Harmonic Analysis. Cham: Birkhäuser (ISBN 978-3-319-65321-1/hbk; 978-3-319-65322-8/ebook). xxi, 359 p. (2018).

This book covers the topics mentioned in its title (where operator theory means basic results on operators on normed, Banach and Hilbert spaces) in a way that is suitable for upper-level courses or for independent study. It requires basic knowledge in calculus, linear algebra, real analysis and a familarity with proofs. In roughly 30 pages, the author however gives at the beginning a "crash course" starting from set theory and covering those facts of analysis and linear algebra that will be needed in the following chapters.

Of particular interest is the way the content is organized. The author suggests himself different courses that can be taught with this book depending on interests and prerequisites of the students: A first option is a course on metric, Banach and Hilbert spaces that starts with going carefully through the aforementioned preliminiaries section and then focusing on the different types of spaces and results on their structure. The second option is to go much faster through these chapters, possibly skipping the preliminaries. In turn it is then possible to cover results on operators between Banach spaces (e.g., the fact that $L(X,Y)$ is Banach for X, Y Banach, dual spaces, the uniform boundedness principle) and between Hilbert spaces (e.g., the definition of adjoints, the spectral theorem for compact self-adjoint operators). Both of these courses – which are precisely outlined in the introduction – leave at the end of each chapter several topics to the interested student. These topics can of course also be covered in the classroom. The author recommends a 2-semester course if all topics shall be covered.

Each subchapter is endowed with a number of exercises. These vary in difficulty and type. Very interesting, for the classroom as well as for independent study, is the combination of more computational problems ("show that d is a metric, show that $x \in \ell^2$ but $y \notin \ell^2$"), the usual proof-oriented problems and problems that help students to think about definitions and theorems from the main text ("explain why $\mathbb{R}^d$ is the same as $\{f:\{1,\dots,d\} \to \mathbb{R}\}$, find a space X such that whatever holds, can one prove Theorem A also without using Lemma B?").

To sum up, this book can be used for designing courses that start with topics covered in a high level real analysis course (such as metric spaces, convergence, completeness, open and closed sets) but ends with elements that usually appear in introductory courses on functional analysis (such as spaces of operators, the uniform boundedness principle, dual spaces, adjoints). The book here allows a very flexible design, taking into account the prerequisites of the students and the instructor's ambitions.

Reviewer: Sven-Ake Wegner (Middlesbrough)

MSC:

46-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to functional analysis
47-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to operator theory
54-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to general topology
00A05	Mathematics in general

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Banach space; Hilbert space; operator; metric space

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