**ERRATA TO:**

# LINEAR INDEPENDENCE OF FINITE GABOR SYSTEMS

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**Note:** This errata is not included in the published version of this paper.

(1) On page 181, the sentence

> One technical point is that the definition of basis really combines aspects of both spanning and independence, i.e., a basis is necessarily complete and has Property S.

should be replaced by

> One technical point is that the definition of basis really combines aspects of both spanning and independence, i.e., a basis is necessarily complete and minimal.

(2) In Theorem 9.9, replace the hypothesis

> "Let $\{f_i\}_{i\in\mathbf{N}}$ be a countable sequence of elements in a Hilbert space $H$"

by the hypothesis

> "Let $\{f_i\}_{i\in\mathbf{N}}$ be a frame for a Hilbert space $H$."

(3) The statement of Theorem 9.20 is correct; however, the proof given for part (b) of Theorem 9.20 is incorrect. Replace with the following proof.

*Proof.* (b) Let $A$, $B$ be frame bounds for $\mathcal{G}(g,\Lambda)$ as a frame for its span. Fix $0 < \delta < A^{1/2}/(2N^{1/2})$. Then by (9.19), we can choose $\varepsilon$ small enough that for all $|r| \le \varepsilon$ we have

$$\|T_r g - g\|_2 \le \delta \qquad \text{and} \qquad \|M_r T_{\alpha_k} g - T_{\alpha_k} g\|_2 \le \delta,\ k = 1, \dots, N.$$

Now suppose that $|\alpha_k - \alpha'_k| < \varepsilon$ and $|\beta_k - \beta'_k| < \varepsilon$ for $k = 1, \dots, N$. Then for any scalars $c_1, \dots, c_N$ we have

$$\begin{aligned}
\Big\|\sum_{k=1}^N c_k\, M_{\beta_k} T_{\alpha_k} g\Big\|_2 &\le \Big\|\sum_{k=1}^N c_k\,(M_{\beta_k} - M_{\beta'_k}) T_{\alpha_k} g\Big\|_2 \\
&\qquad + \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k}(T_{\alpha_k} - T_{\alpha'_k}) g\Big\|_2 + \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g\Big\|_2 \\
&\le \sum_{k=1}^N |c_k|\, \|M_{\beta_k - \beta'_k} T_{\alpha_k} g - T_{\alpha_k} g\|_2 \\
&\qquad + \sum_{k=1}^N |c_k|\, \|T_{\alpha_k - \alpha'_k} g - g\|_2 + \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g\Big\|_2 \\
&\le 2\delta \sum_{k=1}^N |c_k| + \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g\Big\|_2 \\
&\le 2\delta N^{1/2} \bigg(\sum_{k=1}^N |c_k|^2\bigg)^{1/2} + \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g\Big\|_2.
\end{aligned}$$

However, we also have by Lemma 9.19 that

$$A^{1/2} \bigg(\sum_{k=1}^N |c_k|^2\bigg)^{1/2} \le \Big\|\sum_{k=1}^N c_k\, M_{\beta_k} T_{\alpha_k} g\Big\|_2.$$

Combining and rearranging these inequalities, we find that

$$\Big(A^{1/2} - 2\delta N^{1/2}\Big) \bigg(\sum_{k=1}^N |c_k|^2\bigg)^{1/2} \le \Big\|\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g\Big\|_2.$$

Since $A^{1/2} - 2\delta N^{1/2} > 0$, it follows that if $\sum_{k=1}^N c_k\, M_{\beta'_k} T_{\alpha'_k} g = 0$ a.e., then $c_1 = \cdots = c_N = 0$. $\square$

## Acknowledgment

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