A COMPENDIUM OF PROBLEMS

I. Find a formula for u if $u'' = f$ and
(a) $u(0) = u(1) = 0$.
(b) $u(0) = u'(0) = 0$.
(c) $u(0) = 3$, $u(1) = 5$,
(d) $u'(0) = 3u(0)$, $u'(1) = 5\,u(1)$,
(e) $u(0) = u(1)$, $u'(0) = u'(1)$,
(f) $u(0) = 3$, $u'(0) = 5$,
(g) $u'(0) = 3$, $u(0) = 5$,
(h) $u(0) = 0$, $\int_0^1 u(x)\,dx = 0$.

II. Find a formula for u if $u'' + 9u = f$ and
(a) $u(0) = 3$, $u'(1) = 5$.
(b) $u(0) - u'(0) = 3$, $u(1) = 5$.

III. Find a formula for u if $(x\,u'(x))' = f$ and $u(1) = 0$, $u(2) = 5$.

IV. (a) Find conditions on f in order that $u'' + 4\pi^2 u = f$, $u(0)=u(1)$, $u'(0) = u'(1)$ should have a solution.
(b) Give the Green's function for this problem.
(c) By finding the Green's function for the problem $L(y) = y''$, $y(0) = y(1)$, $y'(0) = y'(1)$, re-write this equation as an integral equation such as was studied in the previous chapter.

V. Here is a linear differential operator with boundary conditions:

$$L(y)(x) = (e^x\, y')' \quad \text{and } B_1(y) = y(0),\ B_2(y) = y'(0).$$

A. Show that $(e^x\, y')'\, z - y\, (e^x z')' = [\, e^x\, (z\, y' - z'\, y)]'$.
B. Give L^* and B^*.
C. Give the Green's function for the problem $L(y) = f$ with $B_1(y) = B_2(y) = 0$.
D. Rewrite the problem $(e^x\, y')' + \sin(x)\, y(x) = f(x)$, $y(0) = y'(0) = 0$ as an integral equation in the form

$$y = K(y) + F.$$

Be sure to identify K and F carefully.

VI. Consider the differential equation: f is continuous on $[0,\pi]$ and

$$(\sin(x)\, y'(x))' + 2\sin(x)\, y(x) = f(x)$$

$$y(0) = 0 = y(\pi).$$

A. In the context of this course, what is the appropriate space and linear operator L?
B. What is the adjoint of L in this space? Explain your answer.
C. Is this problem 1st or 2nd alternative?

D. If possible, solve this problem with f(x) = x. If it is not possible, explain why not.

VII. Find a formula for u if

$$u'' + \sin(x)\, u = f,\ u(0) = 3,\ u(1) = 5.$$

ANSWERS

1.(a) $(x-1) \int_0^x t\, f(t)\, dt + x \int_x^1 (t-1)\, f(t)\, dt,$

(b) $\int_0^x (x-t)\, f(t)\, dt,$

(c) $(x-1) \int_0^x t\, f(t)\, dt + x \int_x^1 (t-1)\, f(t)\, dt + 5x - 3(x-1),$

(f) $\int_0^x (x-t)\, f(t)\, dt + 3 + 5x,$

(g) $\int_0^x (x-t)\, f(t)\, dt + 5 + 3x.$