CLASS NOTES FOR

MATH 4348:

AN INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

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PREFACE

Welcome to Math 4348--An Introduction to Partial Differential Equations.

This course is about linear functions L and solving linear equations L(u) = f where f has been previously specified. Students who come into this class are already familiar with equations of this type. For example, if L is an nxn matrix and f is an n-dimensional vector, then an n-dimensional vector u can be found such that L(u) = f if the determinant of L is not zero. If the determinant of L is zero, there are some f's for which there is a solution and some for which there is none. This situation is typical even when the equation is not a matrix equation and when there is no determinant defined on the linear function L. The existence of such L's and f's provokes a reasonable pair of questions for linear equations:

(1) How can one determine whether there are solutions to L(u) = f for all given f's?

(2) In case there are not solutions for all f's, how can one characterize those f's for which there is a solution.

We will answer these two questions for classes of integral equations, classes of ordinary differential equations, and classes of partial differential equations. When possible, the inverse of L, denoted in the usual manner by L^{-1}, will be created. This defines the Green function.

Students planning to take this course often ask at least these three questions: What do I need to know to understand the course? What will be required of me? and, What will be the pace through the lecture notes? Here are some answers.

One should recall a little elementary geometry of R^n: that perpendicularity is determined by the dot product and not by visual inspection, and that the matrix A and its adjoint A* are also related through this dot product:

$$< Ax, y > = < x, A^*y >.$$

In addition to knowing about solutions for A(u) = f, the student should know about solutions of A(u) = 0, especially in case the determinant of A is zero. Be reminded that all such u is called the null space of A.

In the section on integral equations, one must recall the notion of a collection of linearly independent vectors or functions. A recurring idea from the calculus is that if

$$y(x) = \int_0^x f(x)\, g(t)\, dt,$$

where f is differentiable and g is continuous, then

$$y'(x) = f'(x) \int_0^x g(t)\, dt + f(x)\, g(x).$$

A review of the differentiation of integrals with variable limits of integration might be advisable.

From an introductory course in ordinary differential equations, one should be able to solve low order, constant coefficient differential equations. For example, one would be expected to choose the pair cos(3x) and sin(3x) as solutions of $y'' + 9y = 0$, instead of the pair e^{3ix} and e^{-3ix}. While both are technically correct, there are many advantages to using the first pair in the course.

In the last section of these notes, the multi-dimensional calculus is used. One should feel comfortable computing gradients and Laplacians of smooth functions and computing normals to simple surfaces in R^3. The divergence theorem in R^2 and R^3 is fundamental in computing the adjoints of some to the linear functions of that section.

There will be computer projects for this course. They should be done in the course for they show the utility of the methods developed. Maple syntax and output will be suggested for the projects. Additionally, lecture notes will use this software for computation and visualization of related ideas. Georgia Tech has a Maple site license for university owned machines. Additionally, Maple is included in the software package which incoming students purchase.

The pace through the notes will be to use about one third of the time for each of the chapters -- chapters on integral equations, ordinary differential equations, and partial differential equations. It is hoped that the future editions of these notes will be improved because of your suggestions. Thanks are expressed to previous classes who have provided corrections to previous editions and answers to exercises.