Addendum for Modules 15 through 20:

Cooling: Point Source or Radiating Rod

Radiating Rod: $u_t = k\, u_{xx} - c\,(\, u(t, x) - 32\,)$

$u(t, 0) = 32 = u(t, 1)$

$u(0, x) =$ specified.

Newton's Law of Cooling:

$T' = \alpha\,(\, T(t) - 32\,)$

$T(0) =$ specified.

Solution for Radiation Cooling of Radiating Rod:

$$u(t,x) = 32+\exp(-c\ t)\ \Sigma\ A_p \sin(p\ \Pi\ x)\ \exp(-k\ p^2\Pi^2\ t).$$

Solution for Newton's Law for a Point Source

$$T(t) = 32 + \exp(-\alpha\ t)\ (T(0) - 32).$$

Here is the issue I bring to this lecture:

Given a rod with $k = 1/10$, $c = 1/50$, and

$u(0, x) = 32 + 272\, x\, (1 - x)$.

We know

$$u_t = k\, u_{xx} - c\, (\, u(t, x) - 32\,)$$
$$u(t, 0) = 32 = u(t, 1)$$

Is there α ?

Solve the diffusion equation with radiation cooling.

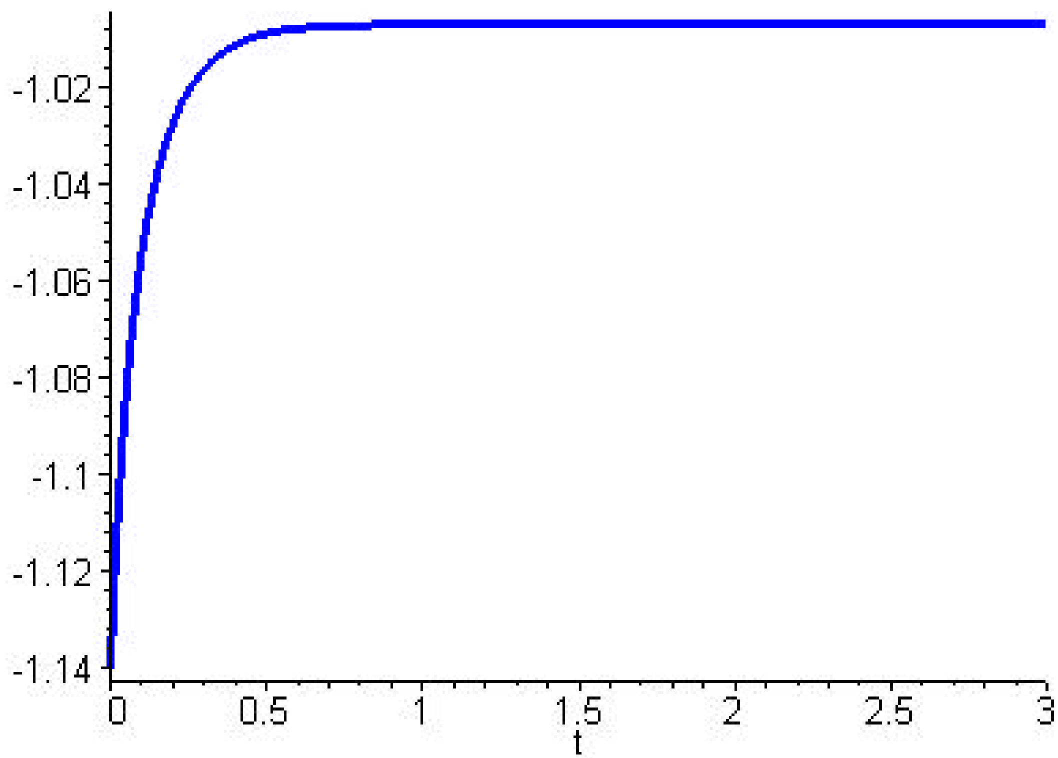
Take

$$T(t) = \int u(t, x)\, dx$$

Is there α such that

$$T' = \alpha\, (T(t) - 32)$$
$$\text{with } T(0) = \int u(0, x)\, dx$$

To answer this question, we compute T(t) and draw the graph of T'/(T(t) - 32). Is this constant?



This lecture was presented for my benefit. I have taught

(a) Newton's Law of Cooling to Freshmen, and

(b) Diffusion, with Radiation Cooling to you.

Reconcile.

Enough? No! Why not:

! Experimental Mathematics !