# Addendum Module 9:

## Convergence of Series, with Examples

This lecture is a summary:

four facts about convergence,

three examples to illustrate the facts.

Series Model: $\sum a_p \sin(p x)$.

Fact 1. The Series Model converges in norm if

and only if the number sequence $\{ a_1, a_2, a_3, \ldots \}$

has the property that $\sum | a_p |^2$ is finite.

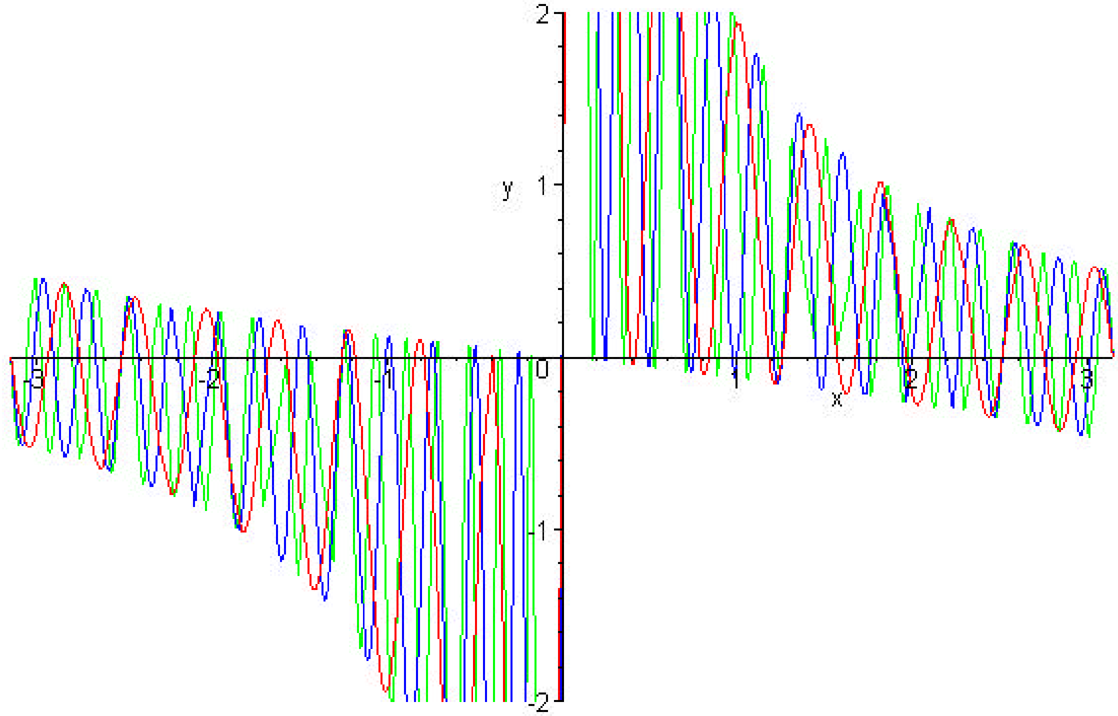
Thus,

$$\sum \sin( p\, x)$$

does not converge in norm, and

$$\sum \sin( p\, x)/p$$

does converge in norm.



2
y
1
-3
-2
-1
0
1
x
2
3
-1
-2

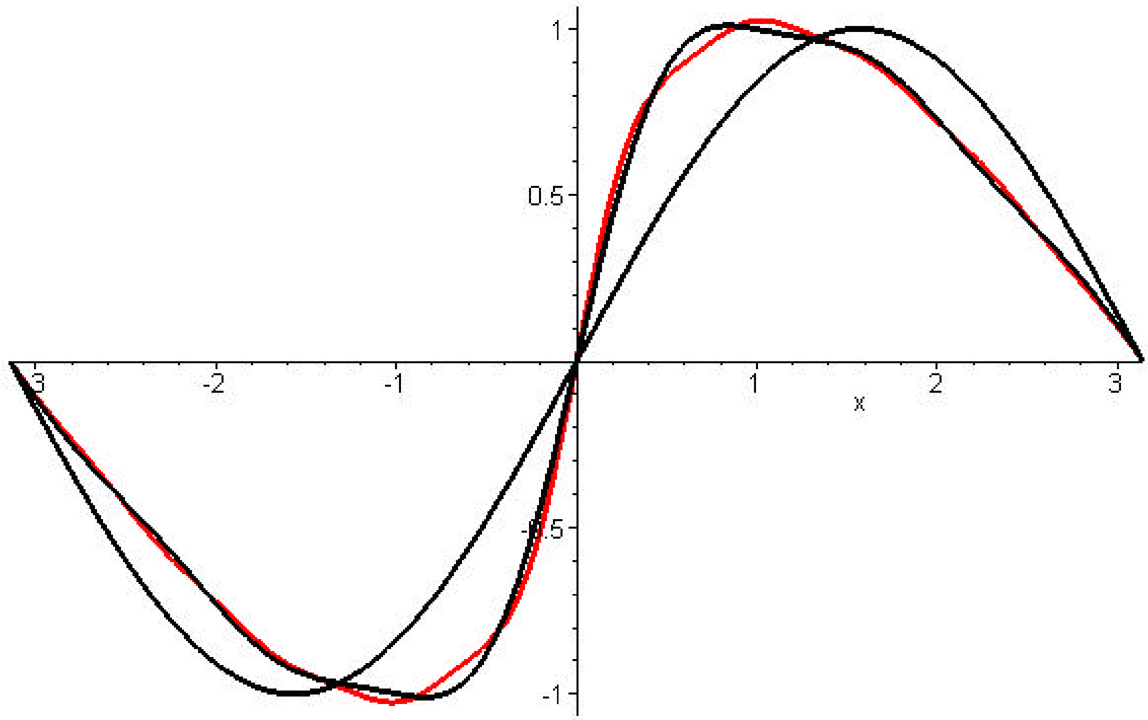
Fact 2: If the number sequence has the property

that $\sum |a_p|$ is finite then the series model

converges uniformly.

Here's why: Max $|\sum a_p \sin(p x)| \le |\sum a_p|$

Example: $\sum 1/p^2 \sin(p x)$ .



1
0.5
-0.5
-1
-3
-2
-1
1
2
3
x

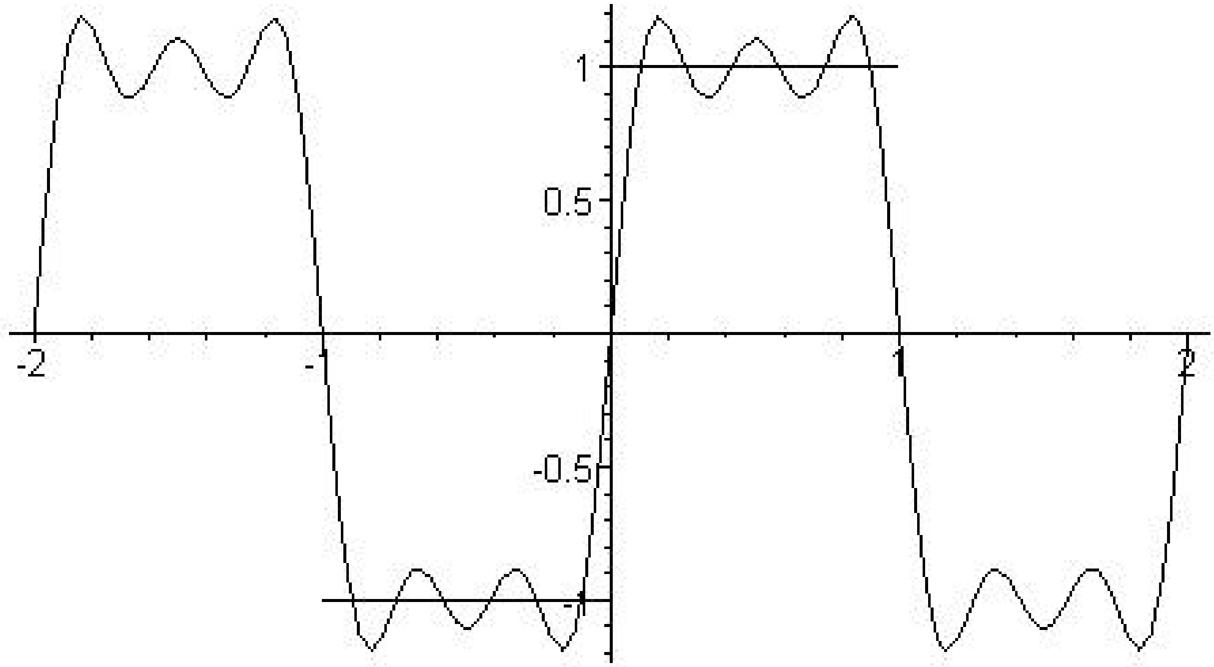
Fact 3: (about pointwise convergence) If the limit function is sectionally smooth and periodic with period $2\Pi$, then the series model converges at every x. The limit is

$$1/2\ [f(x+) + f(x-)].$$

Consider the function signum(x) restricted to $[-\Pi, \Pi]$ and extended periodically. To what does the function converge at $x = 0$?

Fact 4: If the limit function is not continuous, the series model does not converge uniformly.

Consider the function signum(x) restricted to $[-\Pi, \Pi]$ and extended periodically. The Fourier series does converge pointwise but not uniformly.



1
0.5
-2
-1
1
2
-0.5
-1

Why did I make this addendum?

If you are given a function, you might ask: Does the series converge uniformly?

We have a criteria for no.

We have a criteria for yes.

I do not have a necessary and sufficient condition.