Welcome to Math 4581:

Classical Mathematical Methods in Engineering

Course topics:

Fourier Series,

Boundary Value Problems for Partial Differential Equations, and

the Laplace Transform.

Recommended Readings for MATH 4581:

Electronic Text located at
http://www.mathphysics.com/pde/

Partial Differential Equations and
Boundary Value Problems with Maple
George Articolo, Academic Press, ISBN 0-12-064475-4

Introduction to Partial Differential Equations
with MATLAB
Jeffrey M. Cooper, Birkhauser, ISBN 0-8176-3697-5

Boundary Value Problems
David L. Powers, HBJ, ISBN 0-15-505535-6

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Module 1: Linear Spaces

Definition: A linear space of functions is a collection of functions all having the same domain and

(a) if each of f and g belongs to the collection, then f + g does also, and

(b) if f is in the collection and r is a number, then r f is in the collection.

Examples: two of these collections are not linear spaces:

(1) C([0,1]).

(2) R^3.

(3) points in R^3 with last component zero.

(4) points in R^3 with first component one.

(5) points {x, y, z} in R^3 such that

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

(6) functions with period 2π.

(7) positive functions on [0,1].

(8) functions f in C([0,1]) with f(0) = f(1) = 0.

Definition: A basis for a vector space is a collection of linearly independent vectors { x_1, x_2, ... , x_n} such that any vector in the space can be written as a linear combination of these.

Examples:

(1) { [1, -1, 0], [1, 1, 0], and [0, 0, 1] } is a basis for R^3.

(2) {1, x, x^2, ...} is a basis for the polynomials.

(3) { [1, -1, 1], [1, 1, 1], and [1, 0, 1]} is not a basis for R^3.

Definition: If S is a collection of vectors, then the span of S is all vectors that can be written as finite linear combinations of S.

Example:

The span of { [1, -1, 1], [1, 1, 1], and [1, 0, 1]} is all vectors of the form { [x, y, x]}

Definition: A <u>linear operator</u>, say L, is a function with domain a vector space and for which

$$L(a x + y) = a L(x) + L(y).$$

The <u>nullspace</u>, or kernel, for a linear operator is all vectors x such that L(x) = 0.

The <u>range</u> of a linear operator is all vectors y for which there is an x so that L(x) = y.

We find the kernel and basis for the kernel using the linear differential operators

$$L(y) = y'' - y' - 2y$$

and

$$L(y) = y'' + y' + 2y.$$

Hint: $r^2 - r - 2 = 0$ has real roots

and

$r^2 + r + 2 = 0$ has complex roots.

Home work: See Maple Worksheet for Module 1

Module 1 Summary:

A linear space of functions

A basis for a space

A span for a collection of vectors

A linear operator, its nullspace and range