Module 6: Examples: Computations and Graphs

The set $\{1, \cos(\pi x/L), \cos(2\pi x/L), \cos(3\pi x/L), \ldots\}$ is orthogonal in $C([0, L])$. If f is in $C([0, L])$ then

$$f = \sum_n a_n \cos(n\pi x / L),$$

where

$$a_n = < f, \cos(n \pi x/L)>/< \cos(n \pi x/L), \cos(n \pi x/L)>.$$

Here, if $n > 0$,

$$< f, \cos(n\pi x/L)> = \int_0^L f(x)\cos(n\pi x/L)\,dx$$

and

$$< \cos(n\pi x/L), \cos(n\pi x/L)> = \|\cos(n\pi x/L)\|^2 = 2/L$$

$$a_0 = 1/L \int_0^L f(x)\,dx.$$

The set $\{\sin(\pi x/L), \sin(2\pi x/L), \sin(3\pi x/L), \ldots\}$ is orthogonal in C([0, 1]). If f is in C([0, 1]) then

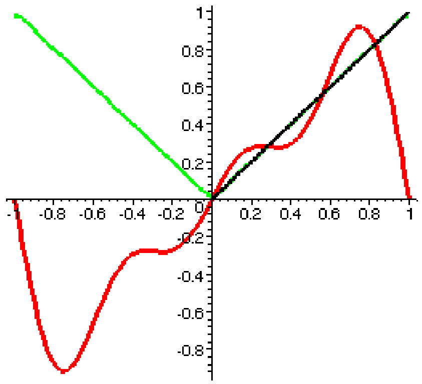
$$f = \sum_n a_n \sin(n\pi x/L),$$

where

$$a_n = < f, \sin(n \pi x/L)>/< \sin(n \pi x/L), \sin(n \pi x/L)>.$$

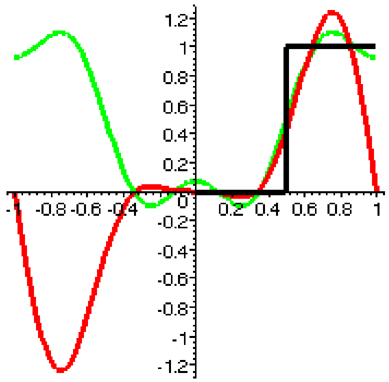
Here are three terms of the two series on [0,1].

The function: f(x) = x.



Here are three terms of the two series on [0,1].

The function: f(x) =Heaviside(x-1/2)



Recall from Module 3 that we need the infinite collection of sine and cosine functions for the symmetric interval $C[-L, L]$

$$\{\cos(n\pi x/L), \text{ for } n = 0, 1, 2, \ldots \text{ and}$$

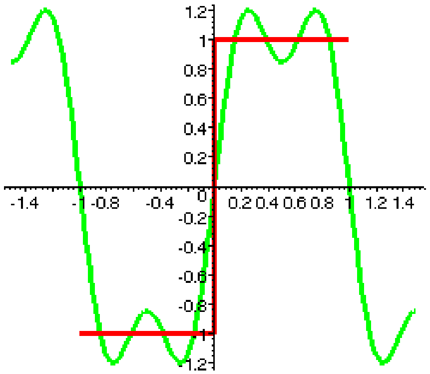
$$\sin(n\pi x/L) \text{ for } n = 1, 2, 3\}.$$

The structure for the Fourier Coefficients is the same, but the dot product is defined by

$$\langle f, g\rangle = \int_{-L}^{L} f(x)g(x)dx.$$

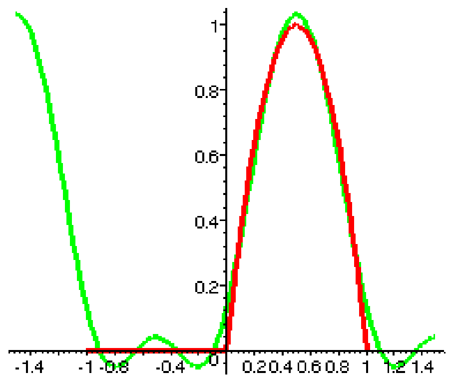
Here are three terms of the series on [-1,1].

The function: f(x) = 2 Heaviside(x)-1.



Here are three terms of the series on [-1,1].

The function: f(x) = 2 x (1-x) Heaviside(x)



Assignment: See the Maple Worksheet

In this Module 6, we have

1. computed Fourier Series on [0,L], and
2. computed Fourier Series on [-L, L].