Module 8: General Convergence

We discuss three types of convergence in C([0, 1]): normed, pointwise, and uniform.

Suppose we have a sequence of functions $f_1(x)$, $f_2(x)$, $f_3(x)$, ... converging to a function g(x). We say that the f's converge to g in the sense of

Norm Convergence if

$$\int_0^1 (f_n(x) - g(x))^2 dx \to 0 \text{ as } n \to \infty .$$

Pointwise Convergence if, for each x,

$f_n(x) \rightarrow g(x)$ as $n \rightarrow \infty$.

Uniform Convergence if the maximum for all x in [0, 1] of the difference in $f_n(x)$ and $g(x)$ goes to zero as $n \rightarrow \infty$: $\max_{x\ [0,1]} | f_n(x) - g(x) | \rightarrow 0$

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1. Uniform Convergence implies pointwise convergence. To see this, note only that if

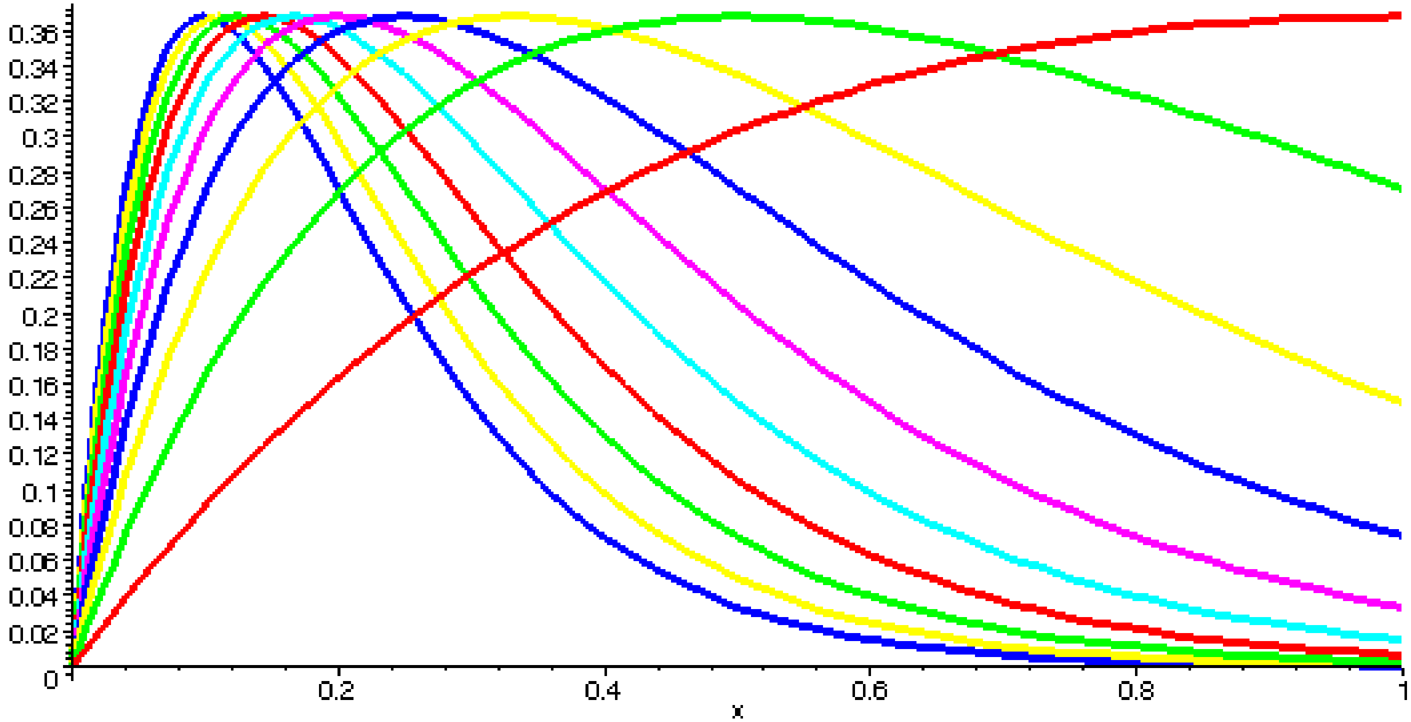
$$\max_{x \ [0,1]} | f_n(x) - g(x) | \to 0$$

then for each x, $| f_n(x) - g(x) | \to 0$.

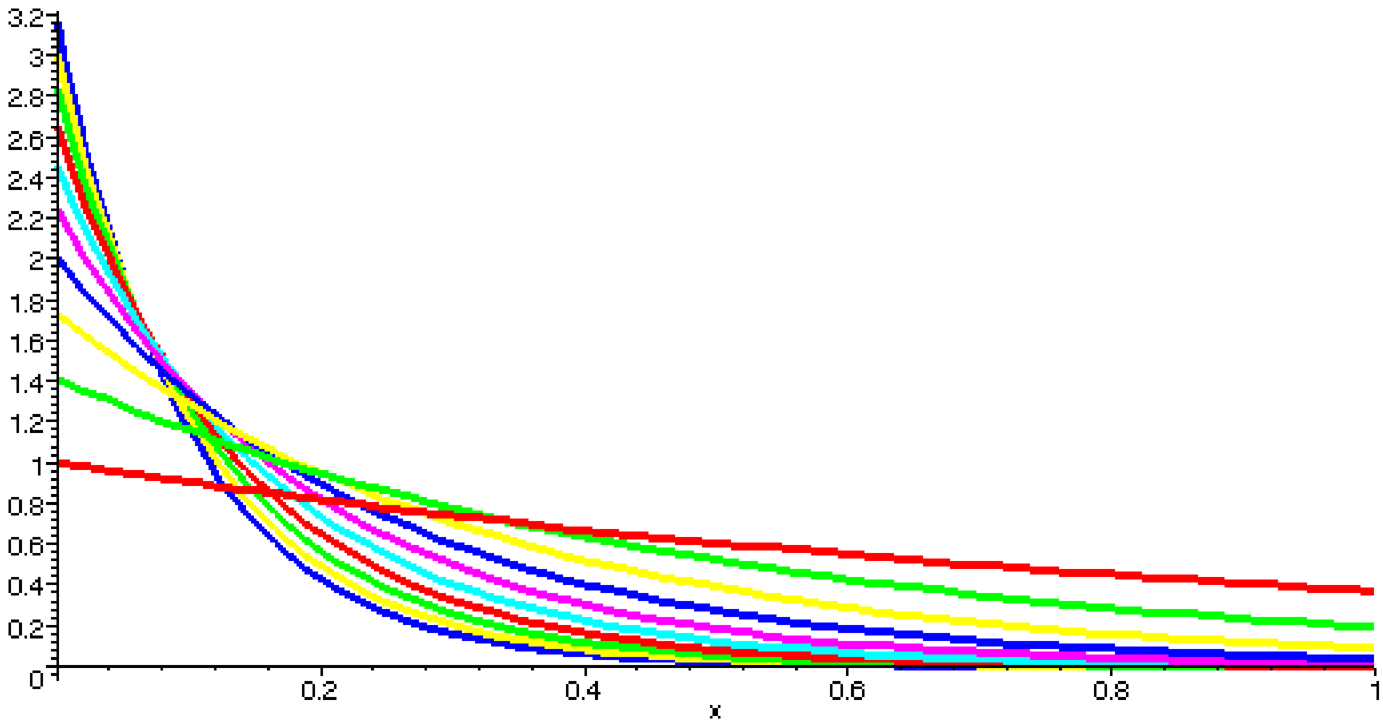
These methods of convergence can be contrasted.

2. Uniform Convergence implies normed convergence. To see this, note that

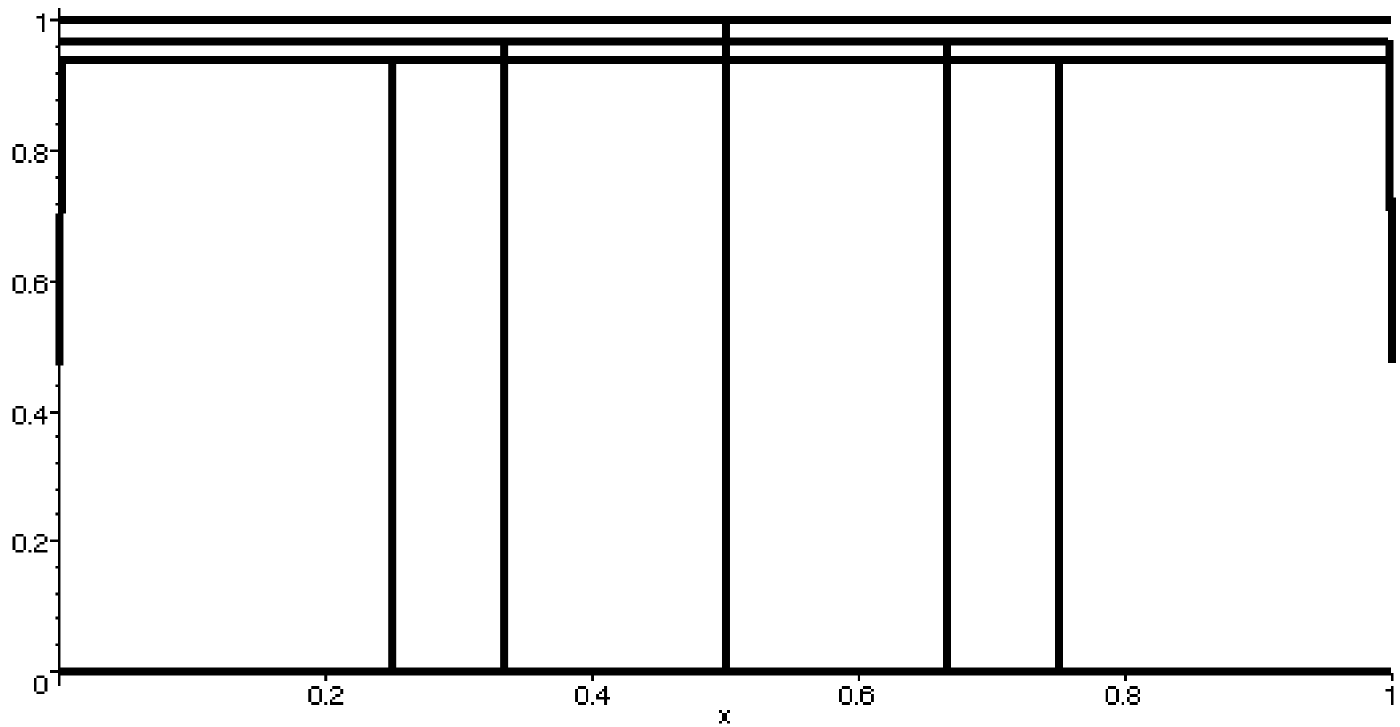
$$\left| \int_0^1 (f_n(x) - g(x))^2 dx \right| \leq \max_{x \ [0,1]} | f_n(x) - g(x) |^2$$



3. Pointwise convergence does not imply uniform convergence. Each Max = 1/e



4. Pointwise convergence does not imply normed convergence.



5. Norm convergence does not imply pointwise convergence.

Assignment: See the Maple Worksheet

In this Module 8, we have

1. Discussed three general types of convergence in C([0,1]).
2. Contrasted these methods of convergence to determine which are the stronger, which are weaker.