# Lecture 20.5: Comparisons and Contrasts

**Examples: The simple diffusion equation, with a variety of boundary conditions.**

**Why is this lecture here?**

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \text{ , } u(0,x) = f(x)$$

$$u(t,0) = 0 = u(t,1)$$

$$X'' = \lambda X, \; X(0) = 0 = X(1),$$

$$T' = \lambda T$$

$$X(x) = \sin(n \pi x),$$

$$T(t) = \exp(- n^2 \pi^2 t)$$

$$u(t,x) = \Sigma_n a_n X_n(x) T_n(t)$$

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \text{ , } \mathbf{u(0,x) = f(x)}$$

$$u_x(\mathrm{t},0) = 0 = u_x(\mathrm{t},1)$$

$$\mathrm{X''} = \lambda\ \mathrm{X},\ \mathrm{X'(0)} = 0 = \mathrm{X'(1)},$$

$$\mathrm{T'} = \lambda\ \mathrm{T}$$

$$\mathrm{X(x)} = \cos(\mathrm{n}\ \pi\ \mathrm{x}),$$

$$\mathrm{T(t)} = \exp(-\,n^2\ \pi^2\ \mathrm{t})$$

$$\mathrm{u(t,x)} = \Sigma_n\ a_n\ X_n(x)\ T_n(t)$$

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \text{ , } u(0,x) = f(x)$$

$$u(t,0) = 0 = u_x(t,1)$$

$$X'' = \lambda X, \ X(0) = 0 = X'(1),$$

$$T' = \lambda T$$

$$X(x) = \sin((2n - 1)\pi/2 \, x),$$

$$T(t) = \exp(-(2n-1)^2 \pi^2 t/4)$$

$$u(t,x) = \Sigma_n a_n X_n(x) T_n(t)$$

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \text{ , } u(0,x) = f(x)$$

$$u_x(t,0) = 0 = u(t,1)$$

$$X'' = \lambda X, \; X'(0) = 0 = X(1),$$

$$T' = \lambda T$$

$$X(x) = \cos((2n - 1)\pi/2\, x),$$

$$T(t) = \exp(-(2n - 1)^2 \pi^2 t/4)$$

$$u(t,x) = \Sigma_n a_n X_n(x) T_n(t)$$

$$\frac{¶}{¶t} u = \frac{¶^2}{¶x^2} u \text{ , u(0,x) = f(x)}$$

$$u\text{(t,0) = 0, } u\text{(t,1/2) = 1}$$

$$\text{X'' = l X, X(0)= 0 = X(1/2),}$$

$$\text{T' = l T}$$

$$\text{X(x) = sin(n p x/L),}$$

$$\text{T(t) = exp(- } n^2 \text{ p}^2 \text{ t/} L^2 )$$

$$\text{u(t,x) = 2 x+ S}_n \, a_n \, X_n(x) \, T_n(t)$$

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \text{ , } \mathbf{u(0,x) = f(x)}$$

$$\mathbf{u(t,0) = 0,}\ u_x(1) = 1 - u(t, 1)$$

$$X'' = \lambda X, \ X(0)=0, \ X'(1)=-X(1)$$

$$T' = \lambda T$$

$$\tan(\lambda) = -\lambda, \ X(x) = \sin(\lambda x)$$

$$u(t,x) = x/2 + \Sigma_n a_n X_n(x) T_n(t)$$

**Review lesson 19 in this last problem.**

**There are other variations. For example, see Lecture 17 b.**

**We have illustrated a variety of boundary conditions and see that not all solutions are sin(n p x/L).**