Sample Questions for MATH 1554 AP Exam

1. (10 points) Determine whether the statements are true or false.

	true	false
a) If A is a diagonalizable $n \times n$ matrix, then $\text{rank}(A) = n$.	◯	◯
b) The Gram-Schmidt algorithm applied to the columns of an $n \times n$ singular matrix produces a set of vectors that form a basis for $\mathbb{R}^n$.	◯	◯

2. (10 points) Give an example of the following. If it is not possible to do so, write *not possible*.

   (a) A matrix, A, that is the standard matrix for the linear transform $T_A : \mathbb{R}^2 \to \mathbb{R}^2$. T_A first reflects points across the line $x_1 = x_2$, and then projects them onto the x_2-axis.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

   (b) A 3×3 matrix, A, in RREF, $\text{Row}(A)^\perp$ is spanned by $\begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

3. Fill in the blanks.

   (a) A is 3×5 and $\dim(\text{Null}(A)) = 2$. The dimension of the column space of A is $\boxed{}$.

   (b) The maximum value of $Q(\vec{x}) = 10x_1^2 - 7x_2^2 - 4x_3^2$ subject to the constraints $\vec{x} \cdot \vec{x} = 1$ and $\vec{x} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$ is equal to $\boxed{}$.