HOMEWORK 10 (BONUS)

Problem 1: Let H be a Hermitean $n \times n$ matrix. Prove that

a) the two matrices $H \pm iI$ are invertible.

b) the matrix

$$(H+iI)(H-iI)^{-1}$$

is unitary.

Problem 2: Let A be any $n \times n$ matrix and consider the $2n \times 2n$ block matrix

$$\mathcal{B} = \left[\begin{array}{cc} 0 & A \\ A^* & 0 \end{array} \right] \; .$$

a) Show that \mathcal{B} is selfadjoint.

b) Show that \mathcal{B} is similar to $-\mathcal{B}$, i.e., find a $2n \times 2n$ unitary matrix \mathcal{V} such that $\mathcal{V}^*\mathcal{B}\mathcal{V} = -\mathcal{B}$.

c) The eigenvalues of \mathcal{B} are real. What else can you say about them?

Please do problems 32 in Section 5.5. problems 16, 22, 23 36 in Section 5.6 in Strang.

Please turn it in for grading on Thursday April 16.