Math 2552 – Differential Equations **Worksheet 11** (Feb 18, ch4 intro)
Georgia Institute of Technology, Spring 2019 Intro to second order equations

Note: As chapter 4 has not yet been covered in lecture, this is merely an intro to get you thinking about the topic.

1. Solve the second order ODE by first transforming into a system of first order ODE:

a. $6y'' - y' - y = 0$

b. $4y'' - 9y = 0$

c. $y'' + 6y' + 16y = 0$

d. $y'' + 3y' + 2y = 0, \quad y(0) = 2,\ y'(0) = -1$

2. (*optional*) Introduction to the theory of second order ODE:

a. (*review*) What can you say about the existence and uniqueness of a solution to the following IVP?

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \mathbf{x} + \begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix}, \quad \mathbf{x}(t_0) = \mathbf{x}_0$$

b. Write the following IVP as a system of first order equations:

$$y'' + p(t)y' + q(t)y = g(t), \quad y(t_0) = y_0,\ y'(t_0) = y_1$$

c. Based on your answers for a and b, what do you think is the equivalent existence and uniqueness theorem for the equation in part b?

d. After transforming into a system of first order ODE, what is the characteristic equation for $ay'' + by' + cy = 0$? Do the roots (i.e., the eigenvalues) change if you multiply the characteristic equation by a?

e. Suppose you have the equation $ay'' + by' + cy = 0$ as in part d, and that the eigenvalues are distinct, real valued. Find a good choice of eigenvectors, and provide a general solution.