

1. Determine the longest interval in which the given IVP is certain to have a unique, twice differentiable solution:

a. $t(t - 4)y'' + 3ty' + 4y = 2, \quad y(3) = 0, \quad y'(3) = -1$

b. $(\ln t)y'' + \frac{t}{t^2 - 4}y' + y = 0, \quad y(1) = 4, \quad y'(1) = 1$

c. $y'' + t^2y' + \tan(t)y = 2, \quad y(0) = 1, \quad y'(0) = -1$

2. Do the functions $y_1(t)$ and $y_2(t)$ constitute a fundamental set of solutions for the given problem?

a. $y'' - 2y' + y = 0; \quad y_1(t) = e^t, \quad y_2(t) = te^t$

b. $(1 - t \cot t)y'' - ty' + y = 0, \quad 0 < t < \pi; \quad y_1(t) = t, \quad y_2(t) = \sin t$