

1. (*linear algebra review*) Find all eigenvalues and eigenvectors: $\begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & 4 \\ 3 & 0 & 1 \end{pmatrix}$

2. Write the following systems in matrix form and state whether they are autonomous or nonautonomous, and homogeneous or nonhomogeneous:

a. $x' = -x + ty, \quad y' = tx - y$

b. $x' = 3x - z, \quad y' = x + y - z, \quad z' = z + t$

3. Show that $\mathbf{x} = \begin{pmatrix} \sin t - t \cos t \\ t \sin t \end{pmatrix}$ is a solution of the initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 2 \sin t \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

4. Transform the 2nd order ODE into a system of first order ODE: $u'' - 2u' + u = \sin t$