PRETEST 1: Duality

NAME: _____

MATH 3406

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Consider $L : \mathbb{R}^3 \to \mathbb{R}^4$ by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 1 Show L is linear.

Problem 2 Find $\mathrm{Im}(L)$ and $\mathcal{N}(L)$. (Draw pictures.)

Problem 3 (important) Classify all subspaces U of $\mathbb{R}^3$ such that

$$\mathbb{R}^3 = \mathcal{N}(L) \oplus U.$$