

PRETEST 2: Duality
NAME: _____

MATH 3406

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Consider $L : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ by

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Remember Problem 3 from PRETEST 1: Classify all subspaces U of \mathbb{R}^3 such that

$$\mathbb{R}^3 = \mathcal{N}(L) \oplus U.$$

Problem 1 What can you say about

$$L|_U : U \rightarrow \text{Im}(L)?$$

Problem 2 Fix standard bases $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for \mathbb{R}^3 and $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ for \mathbb{R}^4 . Find the matrix of L .

Problem 3 Fix standard bases $\{\phi_1, \phi_2, \phi_3\}$ for $(\mathbb{R}^3)'$ and $\{\psi_1, \psi_2, \psi_3, \psi_4\}$ for $(\mathbb{R}^4)'$. Find the matrix of ϕ_j and ψ_k for each $j = 1, 2, 3$ and $k = 1, 2, 3, 4$.