TEST: Duality

NAME: _____

MATH 3406

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If you have difficulty getting started with this, looking back at the associated pretest may help.

$L : \mathbb{R}^3 \to \mathbb{R}^4$ by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 1 Find the matrix of $L' : (\mathbb{R}^4)' \to (\mathbb{R}^3)'$ with respect to the standard bases (and dual bases).

Problem 2 Find the matrix of $T : \mathbb{R}^4 \to \mathbb{R}^3$ by

$$T = \Phi^{-1} \circ L' \circ \Psi$$

with respect to the standard bases where $\Phi : \mathbb{R}^3 \to (\mathbb{R}^3)'$ and $\Psi : \mathbb{R}^4 \to (\mathbb{R}^4)'$ are the standard isomorphisms.