TEST 2: Duality
NAME: ____________

MATH 3406

March 31, 2022

Consider $L : \mathbb{R}^3 \to \mathbb{R}^4$ by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Remember Problem 3 from PRETEST 1: Classify all subspaces U of $\mathbb{R}^3$ such that

$$\mathbb{R}^3 = \mathcal{N}(L) \oplus U.$$

Fix standard bases $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for $\mathbb{R}^3$, $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ for $\mathbb{R}^4$, $\{\phi_1, \phi_2, \phi_3\}$ for $(\mathbb{R}^3)'$, and $\{\psi_1, \psi_2, \psi_3, \psi_4\}$ for $(\mathbb{R}^4)'$.

Fix standard isomorphisms $\Phi : \mathbb{R}^3 \to (\mathbb{R}^3)'$ and $\Psi : \mathbb{R}^4 \to (\mathbb{R}^4)'$.

Problem 1 Find $\mathcal{N}(T)$ and $\mathrm{Im}(T)$ where $T = \Phi^{-1} \circ L' \circ \Psi$.

Problem 2 Find $\mathcal{N}(L')$ and $\mathrm{Im}(L')$.

Problem 3 What can you say about the restriction

$$T\Big|_{\mathrm{Im}(L)} : \mathrm{Im}(L) \to \mathrm{Im}(T)?$$