TEST 3: Duality
NAME: ____________

MATH 3406

March 31, 2022

Consider $L : \mathbb{R}^3 \to \mathbb{R}^4$ by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Remember Problem 3 from PRETEST 1: Classify all subspaces U of $\mathbb{R}^3$ such that

$$\mathbb{R}^3 = \mathcal{N}(L) \oplus U.$$

Fix standard bases $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for $\mathbb{R}^3$, $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}$ for $\mathbb{R}^4$, $\{\phi_1, \phi_2, \phi_3\}$ for $(\mathbb{R}^3)'$, and $\{\psi_1, \psi_2, \psi_3, \psi_4\}$ for $(\mathbb{R}^4)'$.

Fix standard isomorphisms $\Phi : \mathbb{R}^3 \to (\mathbb{R}^3)'$ and $\Psi : \mathbb{R}^4 \to (\mathbb{R}^4)'$.

Problem 1 Given a subspace U of $\mathbb{R}^3$ from Problem 3 of PRETEST 1, what can you say about

$$T \circ L : U \to \mathrm{Im}(T)?$$

Problem 2 If $U = \mathrm{span}\{\mathbf{y}\}$ is a subspace of $\mathbb{R}^3$ from Problem 3 of PRETEST 1 find a formula for

$$T \circ L\mathbf{v} \qquad \text{for} \qquad \mathbf{v} \in U.$$

Problem 3 Find a formula for

$$\left(T \circ L\Big|_U\right)^{-1} \mathbf{w} \qquad \text{for} \qquad \mathbf{w} \in \mathrm{Im}(T).$$