TEST: Homogeneity, Additivity, and Linearity

NAME: ____________________

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Let V and W be vector spaces and consider a function $f : V \to W$.

Problem 1 *Give a precise definition of what it means for f to be* **additive***.*

Problem 2 *Define a function $\phi : \mathbb{C}^2 \to \mathbb{C}^2$ by*

$$\phi(a + bi, c + di) = (a + 2bi, 3c + 4di).$$

(a) *If we consider $\mathbb{C}^2$ as a complex vector space (as usual) what is the dimension of $\mathbb{C}^2$?*

(b) *Show that ϕ is additive.*

Problem 3 *Give a precise definition of what it means for f to be* **homogeneous***.*

Problem 4 *Define a function $g : \mathbb{R}^2 \to \mathbb{R}$ by*

$$g(x, y) = \sqrt[3]{x^3 + y^3}.$$

(a) *Show that g is homogeneous.*

(b) *Show that g is additive on every one-dimensional subspace of $\mathbb{R}^2$, but $g(p + q) \neq g(p) + g(q)$ for any nonzero vectors $p, q \in \mathbb{R}^2$* **not** *in the same one-dimensional subspace.*

(c) *Show the function ϕ from Problem 2 is not homogeneous.*

Problem 5 *Show that if $\dim V = 1$ and $f : V \to W$ is homogeneous, then f is linear.*