

TEST: Quotient and Product Spaces
NAME: _____

MATH 3406

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In a field F there are **products**

$$ab \in F$$

and **quotients**

$$\frac{a}{c} \in F, \quad c \neq 0.$$

Also,

$$\frac{a}{c} c = a.$$

For vector spaces, i.e., in the “category” of all vector spaces over a field F , there are **products**

$$V \times W = \{(v, w) : v \in V, w \in W\}$$

and **quotients**

$$V/Z = \{v + Z : v \in V\} \quad \text{where } Z \text{ is a nonzero subspace of } V.$$

Problem 1 What happens in a quotient of vector spaces when you divide by zero $Z = \{\mathbf{0}\}$?

Problem 2 How about

$$(V/Z) \times Z?$$