Cut and Paste Activity Name and section: ________________

We attempted to cut, paste, and manipulate some strips of paper in order to see that the "lower half" of the projective plane is a Möbius strip. This didn't work out so well (yet). But here are some key assertions/observations intended guide us and keep us on track.

1. The "bottom half" of the projective plane (in the upper hemisphere model) is homeomorphic to a cylinder with one edge antipodally identified and situated in space as indicated in Figure 1(right) (not yet included).

   Figure 1(left) is a hemisphere with a spherical cap removed from the top and a marked included edge and bloody edge to indicate the antipodal identification on the bottom boundary circle. Figure 1(right) is the same strip after we cut it and reattach it to form something like a figure eight.

   Activity: Start with a cylindrical strip (resembling Figure 1(left) including all markings). Convince yourself that manipulation/identification of the bloody edge to match the included edge is not possible without cutting and reattaching. By cutting and reattaching, obtain a strip like the one in Figure 1(right). Convince yourself that this is homeomorphic to the original strip of Figure 1(left).

2. It is now our objective to show that the strip of Figure 1(right) can be (at least in principle) manipulated into a Möbius strip by making the identification required to join the included and bloody edges. More precisely, we claim the strip of Figure 1(right) is what you get if you take a Möbius strip and cut it in half along the middle. Let us call such an object a **cut Möbius strip**.

   There are two practical difficulties in making this manipulation/identification. The first (and most important/interesting) is that it is simply not obvious how to manipulate such a thing around in space in order to make the identifications. That is, it is not obvious how to reassemble a cut Möbius strip. We'll attempt to verify this observation in part 3 of the activity below.

   Second, this particular cut Möbius strip may be two wide to make the manipulation easily

3. (activity) Construct a Möbius strip by taking a strip of paper and taping together the ends with a twist. Cut this strip down the center marking one side of the cut as the bloody edge as you go and the other as the included edge. When you get done, verify that you have a cyliner that matches Figure 1(right).

   See if you can reattach the cut boundary easily to reform the original Möbius strip. Maybe you'll get it, but you should see that there are ways to start which lead to failure, i.e., it's not so obvious how to do it once the cut is made.

   To eliminate this difficulty, repeat this procedure except as you cut also put marks every couple inches to indicate how the cut goes back together (as well as marking the included and bloody edges). Now when you cut you should be able to easily reassemble the cut Möbius strip (cylinder) back into its original uncut position.

   Manipulate between the cut strip and the Möbius strip several times.

   Keep this marked strip.

Name and section: ___________________

4. Repeat the first activity more carefully. Use a narrower strip. First match it to Figure 1(left). Then cut and reattach to get the homeomorphic cylinder of Figure 1(right). Now compare what you've got to your cut Möbius strip and copy the attaching guide marks in appropriate places (and pay attention to the side the mark is on too). Now you should be able to manipulate this cylinder into a Möbius strip by attaching at the matching marks. This gives a verification that the bottom half of the projective plane is a Möbius strip.

5. Write down the homeomorphism as suggested in the quiz and compare to your manipulations.