Math 4431, Assignment 1a: Introduction Name and section: _____

§ 1.1 Armstrong

1. (20 points) Describe two classes of objects for which the Euler characteristic may be defined and which are different from the polyhedral boundaries in Euler's theorem. Make some calculations for these objects and formulate some conjectures.

   Example: The convex hull of the points

   $$(0,0,\ldots,0),\ \mathbf{e}_1=(1,0,\ldots,0),\ \mathbf{e}_2=(0,1,\ldots,0),\ldots,\mathbf{e}_n=(0,0,\ldots,1)$$

   is an object in $\mathbb{R}^n$ with $n+1$ vertices. The Euler characteristic may be defined by

   $$\chi=\sum_{j=1}^{n+1}(-1)^{j+1}\left(\begin{array}{c} n+1 \\ j \end{array}\right)$$

   where the large parentheses denote the calculation of a combination giving the number of $j-1$ dimensional faces. This object is an example of an n **simplex**. The triangle in $\mathbb{R}^2$ has Euler characteristic

   $$\chi=3\text{ (vertices)}-3\text{ (edges)}+1\text{ (face)}=1.$$

   The tetrahedral 3-simplex (solid tetrahedron) in $\mathbb{R}^3$ has

   $$\chi=4\text{ (vertices)}-6\text{ (edges)}+4\text{ (faces)}-1\text{ (volume)}=1.$$

   Conjecture: Every n simplex has Euler characteristic $\chi=1$.