Math 4431, Assignment 1b: Introduction Name and section: ____________________

§ 1.1-6 Armstrong

1. (20 points) (Chapter 1 Problem 6 in Armstrong) A regular polyhedral surface is one in which each face has the same number p edges the same number q faces meeting at each vertex. Use Euler's formula to show
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{2} + \frac{1}{e}.$$

2. (20 points) (Problem 7) Use the previous problem to show there are only 5 regular polyhedral surfaces. Hint: Start by showing $3 \leq p \leq 5$.

3. (20 points) (Problem 10) A **homeomomorphism** is a one-to-one continuous function from one set onto another such that the inverse is also continuous. Show that any two open intervals in the real line $\mathbb{R}$ are homeomorphic. Can you find a one-to-one continuous map of one interval onto another which does not have a continuous inverse?

4. (20 points) (Definition 2.1) Consider the collection $\mathcal{A}$ of subsets A of $\mathbb{R}^n$ with the following property:

   If $p \in A$, then there is some $r > 0$ such that
   $$B_r(p) = \{x \in \mathbb{R}^n : |x - p| < r\} \subset A.$$

   Show that $\mathcal{A}$ is a topology on $\mathbb{R}^n$. (The set $B_r(p)$ is called the ball of radius r centered at p.)