Math 4431, Assignment 2a: open sets Name and section: ________________

§ 2.1 Armstrong

1. (20 points) (2.1.1) Show $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

2. (20 points) (2.1.1) Find sets A and B in a topological space X such that

$$\overline{A \cap B} \neq \bar{A} \cap \bar{B}.$$

3. (20 points) (2.1.3) Let $A = \{(x, \sin(1/x)) \in \mathbb{R}^2 : x > 0\}$.

   (a) Find $\operatorname{int} A$, $\operatorname{clus} A$, $\bar{A}$, and ∂A.

   (b) Consider $X = \bar{A}$ in the subspace topology induced by $\mathbb{R}^2$. Find $\operatorname{int} A$, $\operatorname{clus} A$, $\bar{A}$, and ∂A with respect to X.

4. (20 points) (2.1.7) If X is a topological space and $A \subset X$ is considered in the subspace topology, then show $C \subset A$ is closed in A if and only if $C = A \cap D$ for some closed set D in X.

5. (20 points) Let X be a set. Let

$$\mathcal{T} = \{\phi\} \cup \{A \subset X : A^c \text{ is a finite set}\}.$$

   Show the following:

   (a) $\mathcal{T}$ is a topology on X.

   (b) If $A \subset X$ is infinite, then $\bar{A} = X$.

   (c) If $A \subset X$ is finite, then $\operatorname{clus} A = \phi$.