Math 4431, Assignment 5a: compactness   Name and section: _____

§ 3.1-4 Armstrong

1. (10 points) Show that a metric spae is first countable.

2. (10 points) Show that a separable metric space is second countable. (This is the source of the eroneous problem 1(c) in the last homework.)

3. (10 points) If X is second countable and Y is homeomorphic to X, then Y is second countable.

4. (2.1.11) Let $\mathcal{B} = \{[a, b) \subset \mathbb{R} : a, b \in \mathbb{R}, \ a < b\}$.

   (a) (5 points) Show $\mathcal{B}$ is a base for a topology $\mathcal{T}$ on $\mathbb{R}$. Denote the resulting topological space $(\mathbb{R}, \mathcal{T})$ by X.

   (b) (5 points) Show that every set in $\mathcal{B}$ is closed in X.

5. (10 points) Show that the topological space X from the previous problem is not homeomorphic to $\mathbb{R}$ (with the Euclidean topology).