Math 4431, Assignment 6a: compactness Name and section: ____________________

§ 3.1-4 Armstrong

Definition A topological space X is **locally compact** if each point $x \in X$ has a compact neighborhood, i.e., if there is a compact set N and an open set U with $x \in U \subset N$.

1. (10 points) (3.3.15) Show that $\mathbb{R}^n$ is locally compact.
2. (10 points) (3.3.15) Show that a closed subset of a locally compact space is locally compact. In particular, every closed subset of $\mathbb{R}^n$ is locally compact.
3. (10 points) (3.3.15) The homeomorphic image of a locally compact space is locally compact.
4. (10 points) Is the continuous image of a locally compact space locally compact?
5. (10 points) Find a subset of $\mathbb{R}^2$ which is connected by not path connected.