

Assignment 3 = Exam 1 Fall 2021

Problem 7

In[*]:= `precoeff[L_, j_] = Integrate[(L x - x^2) Sin[j Pi x / L], {x, 0, L}]`

$$\text{Out[*]} = -\frac{L^3 (-2 + 2 \cos[j \pi] + j \pi \sin[j \pi])}{j^3 \pi^3}$$

In[*]:= `coeffseven[L_, M_, j_] = -(2 / L) precoeff[L, j] / Sinh[j Pi M / L]`

`useven[x_, y_, L_, M_, k_] :=`

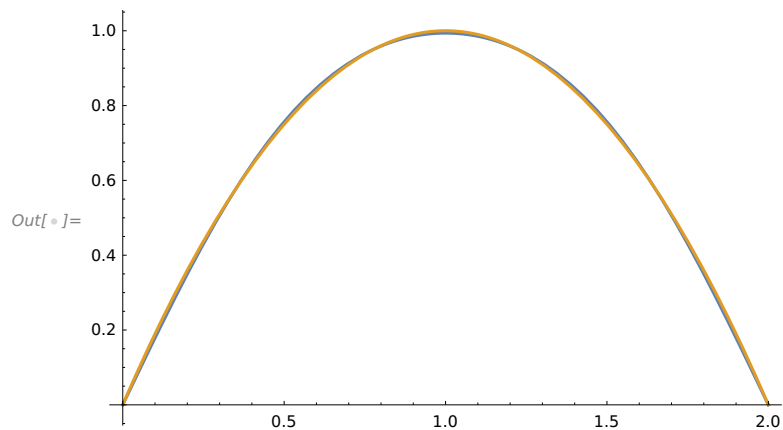
`Sum [coeffseven[L, M, j] Sin[j Pi x / L] Sinh[j Pi (y - M) / L], {j, 1, k}]`

$$\text{Out[*]} = \frac{2 L^2 \text{Csch}\left[\frac{j M \pi}{L}\right] (-2 + 2 \cos[j \pi] + j \pi \sin[j \pi])}{j^3 \pi^3}$$

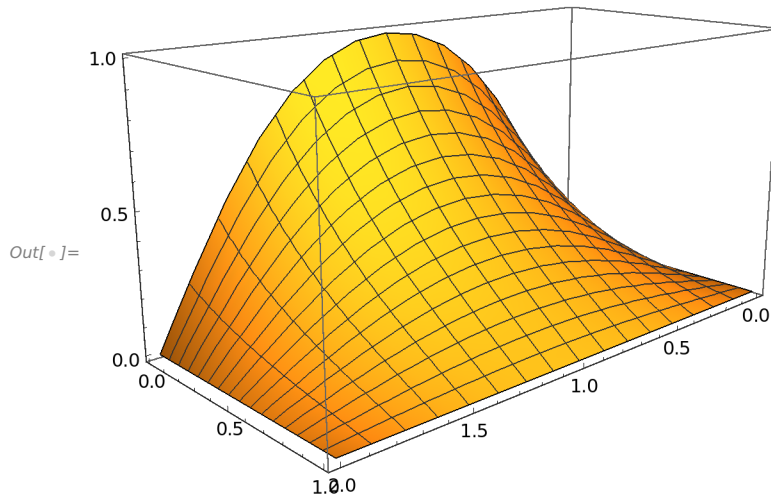
In[*]:= `usevent[x_, y_] = useven[x, y, 2, 1, 3]`

$$\text{Out[*]} = -\frac{32 \text{Csch}\left[\frac{\pi}{2}\right] \sin\left[\frac{\pi x}{2}\right] \sinh\left[\frac{1}{2} \pi (-1 + y)\right]}{\pi^3} - \frac{32 \text{Csch}\left[\frac{3\pi}{2}\right] \sin\left[\frac{3\pi x}{2}\right] \sinh\left[\frac{3}{2} \pi (-1 + y)\right]}{27 \pi^3}$$

In[*]:= `Plot[{usevent[x, 0], 2 x - x^2}, {x, 0, 2}]`



```
In[*]:= Plot3D[usevent[x, y], {x, 0, 2}, {y, 0, 1},
  BoxRatios -> {2, 1, 1}, ViewPoint -> {1.3, 1.4, 0.5}]
```



Problem 8

Piecewise constant g

```
In[*]:= mycoeff[k_] = 8 (-1)^(k + 1) / ((2 k + 1)^2 Sinh[(2 k + 1) Pi / 2] Pi ^ 2)
```

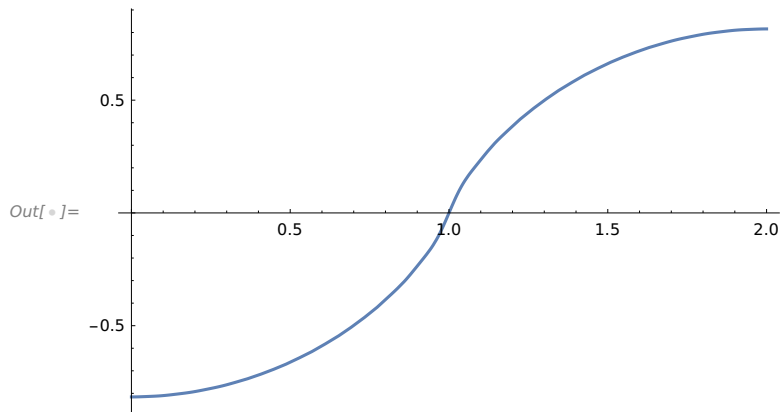
Out[*]=

$$\frac{8 (-1)^{1+k} \operatorname{Csch}\left[\frac{1}{2} (1 + 2 k) \pi\right]}{(1 + 2 k)^2 \pi^2}$$

```
In[*]:= myueight[x_, y_, ell_] := Sum[mycoeff[k] Cos[(2 k + 1) Pi x / 2] Cosh[(2 k + 1) Pi y / 2], {k, 0, ell}]
```

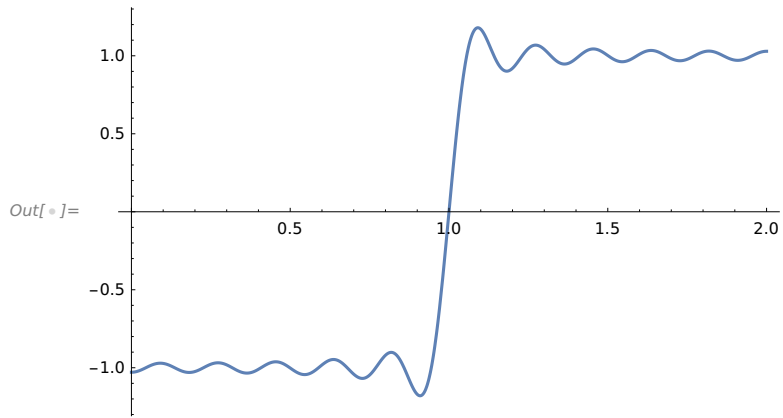
```
In[*]:= myueighty[x_, y_, ell_] :=
  D[Sum[mycoeff[k] Cos[(2 k + 1) Pi x / 2] Cosh[(2 k + 1) Pi y / 2], {k, 0, ell}], y]
```

```
In[*]:= Plot[myueight[x, 1, 20], {x, 0, 2}]
```

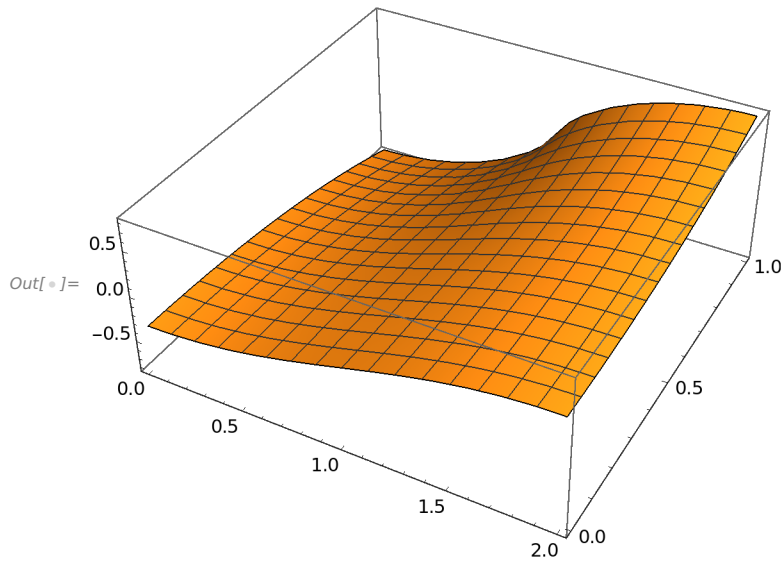


```
In[*]:= evalmyueighty[x_, y_] = myueighty[x, y, 10];
```

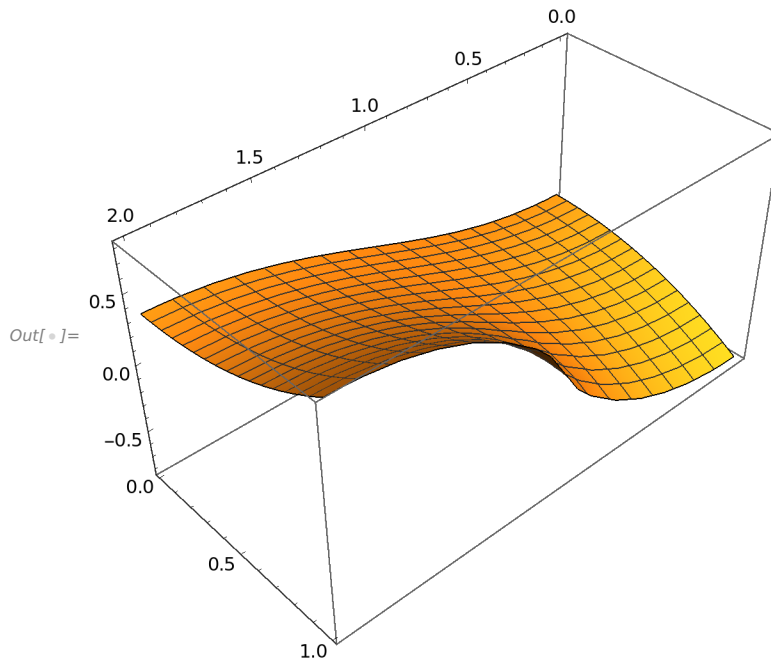
```
In[*]:= Plot[evalmyueighty[x, 1], {x, 0, 2}]
```



```
In[*]:= Plot3D[myueight[x, y, 20], {x, 0, 2}, {y, 0, 1}]
```



```
In[*]:= Plot3D[myueight[x, y, 20], {x, 0, 2}, {y, 0, 1},
  BoxRatios -> {2, 1, 1}, ViewPoint -> {1.3, 1.4, 1.5}]
```



Piecewise linear g (Julian's choice)

```
In[*]:= julianintegral[j_] =
  Simplify[Integrate[(2 x) Cos[j Pi x / 2], {x, 0, 1}] + Integrate[(2 x - 4) Cos[j Pi x / 2], {x, 1, 2}]]
```

$$\text{Out[*]} = \frac{8 \left(-1 + \cos[j \pi] + j \pi \sin\left[\frac{j \pi}{2}\right] \right)}{j^2 \pi^2}$$

```
In[*]:= juliancoeff[j_] = 2 julianintegral[j] / (j Pi Sinh[j Pi / 2])
  ujulian[x_, y_, k_] := Sum[juliancoeff[j] Cos[j Pi x / 2] Cosh[j Pi y / 2], {j, 1, k}]
```

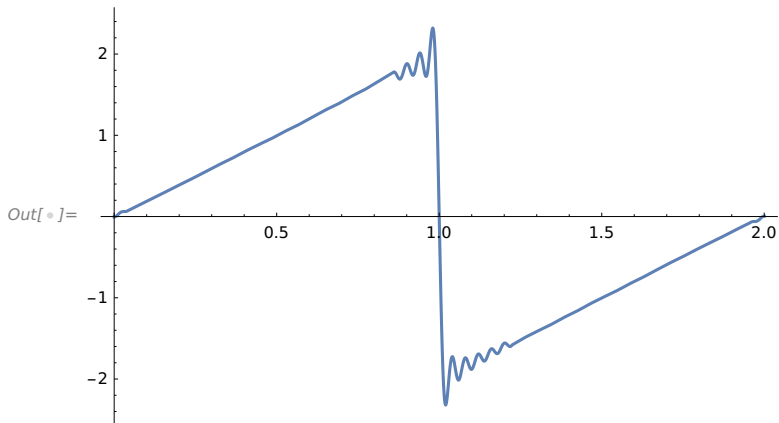
$$\text{Out[*]} = \frac{16 \text{Csch}\left[\frac{j \pi}{2}\right] \left(-1 + \cos[j \pi] + j \pi \sin\left[\frac{j \pi}{2}\right] \right)}{j^3 \pi^3}$$

```
In[*]:= ujulian[x, y, 2]
```

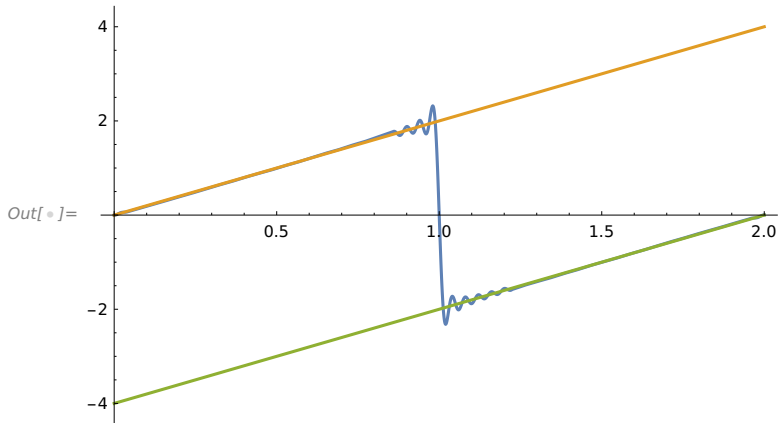
$$\text{Out[*]} = \frac{16 (-2 + \pi) \cos\left[\frac{\pi x}{2}\right] \cosh\left[\frac{\pi y}{2}\right] \text{csch}\left[\frac{\pi}{2}\right]}{\pi^3}$$

```
In[*]:= ujulianeval[x_, y_] = ujulian[x, y, 100];
  uyjulian[x_, y_] = D[ujulianeval[x, y], y];
```

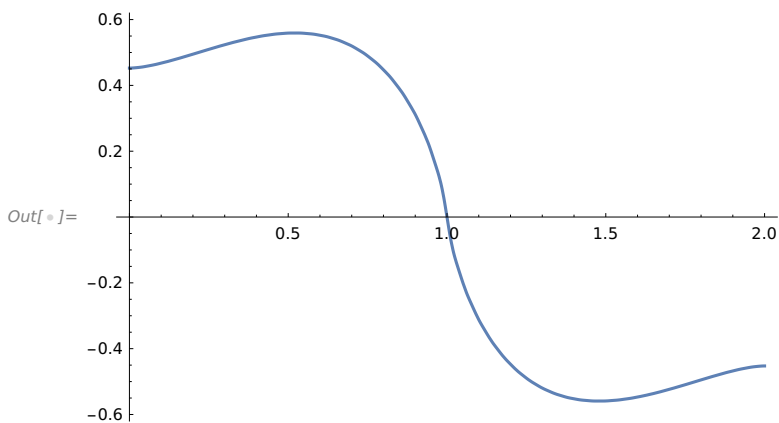
In[*]:= Plot[uyjulian[x, 1], {x, 0, 2}]



In[*]:= Plot[{uyjulian[x, 1], 2 x, 2 x - 4}, {x, 0, 2}]



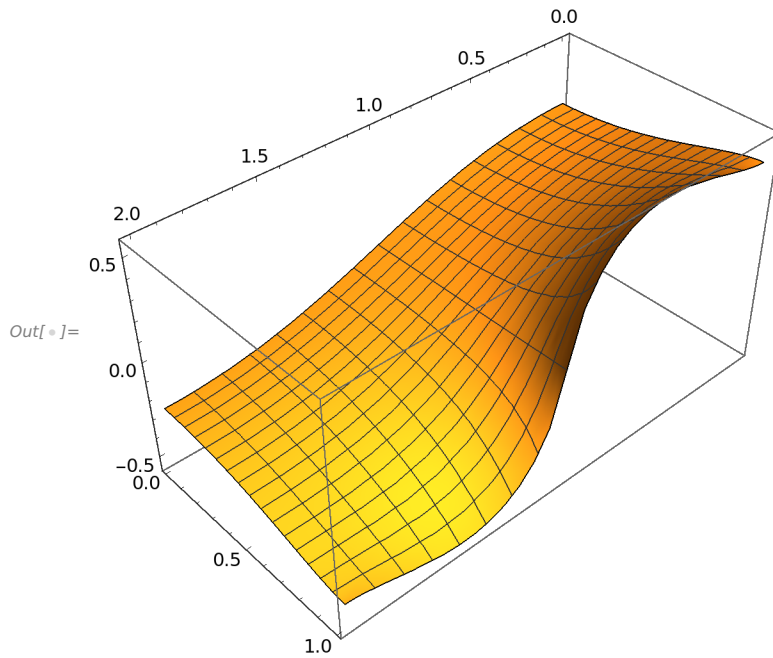
In[*]:= Plot[ujulian[x, 1, 100], {x, 0, 2}]



In[*]:= ujulian[x, y, 1]

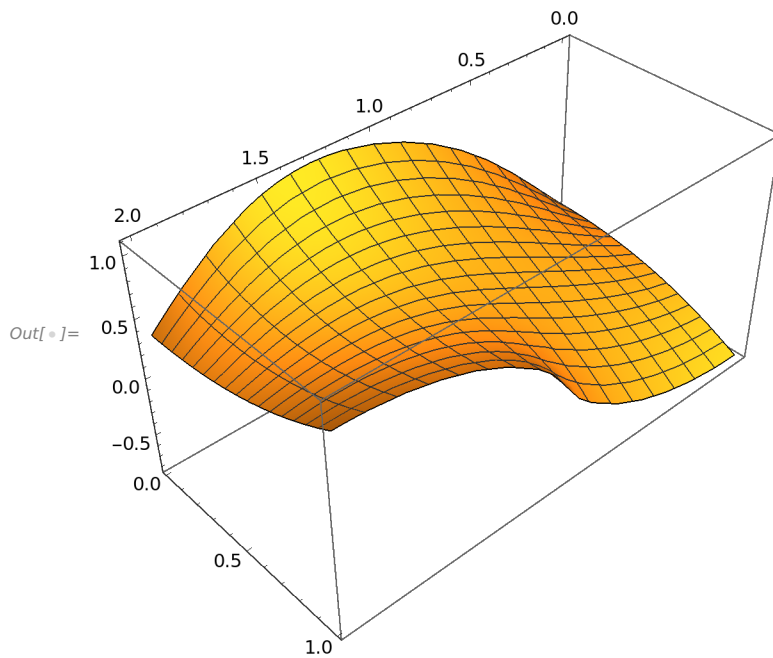
$$\text{Out[*]} = \frac{16(-2 + \pi) \cos\left[\frac{\pi x}{2}\right] \cosh\left[\frac{\pi y}{2}\right] \operatorname{csch}\left[\frac{\pi}{2}\right]}{\pi^3}$$

```
In[*]:= Plot3D[ujulian[x, y, 100], {x, 0, 2}, {y, 0, 1},  
BoxRatios -> {2, 1, 1}, ViewPoint -> {1.3, 1.4, 1.5}]
```



Sum Plots U_7 + U_8

```
In[*]:= Plot3D[usevent[x, y] + myueight[x, y, 20], {x, 0, 2},  
{y, 0, 1}, BoxRatios -> {2, 1, 1}, ViewPoint -> {1.3, 1.4, 1.5}]
```



```
In[ ]:= Plot3D[usevent[x, y]+ujulian[x, y, 100], {x, 0, 2},  
  {y, 0, 1}, BoxRatios -> {2, 1, 1}, ViewPoint -> {1.3, 1.4, 1.5}]
```

