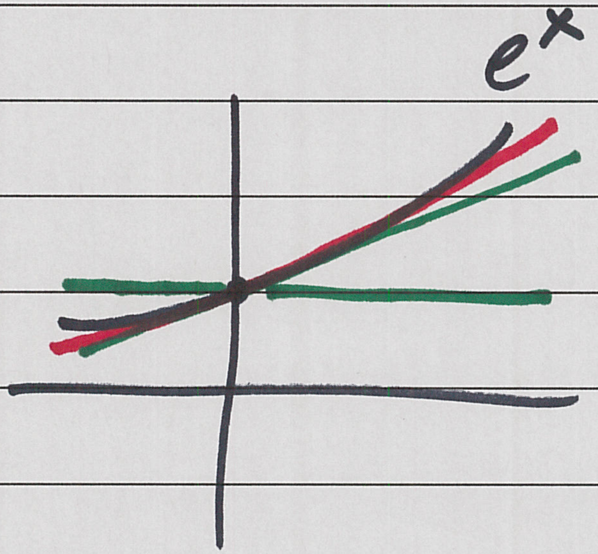
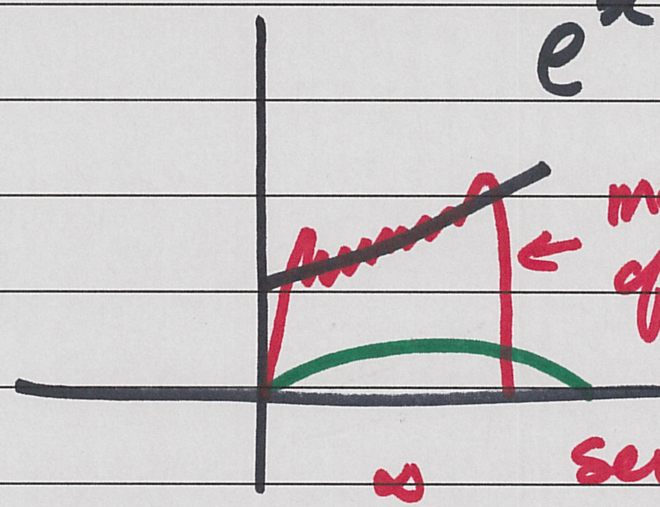
MATH 4581 Tuesday August 31, 2021

Review of last lecture





many terms of the Fourier series

$$\sum_{j=1}^{\infty} a_j \sin(j\pi x)$$

$e^0 = 1$

$\ell(x) = x + 1$

$q(x) = \frac{1}{2}x^2 + x + 1$

[redacted]

Why represent f in terms of Fourier series

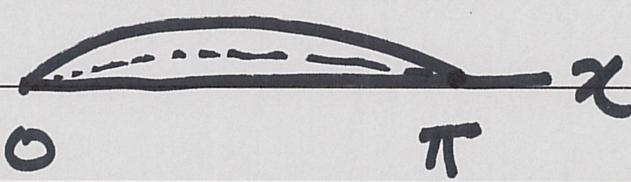
-versus- power series ?

$\boxed{u_t = \Delta u}$)

Fourier knew

$\boxed{u(x,t) = e^{-t} \sin x}$

heat conduction



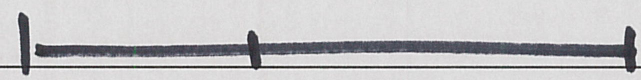
$\boxed{u(x,0) = \sin x}$

Also, $u(x,t) = e^{-j^2 t} \sin jx$ also works.

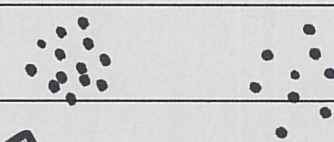
[redacted]

Say we want to model the <u>conduction</u> of heat energy in a thin rod.

(1-D heat equation ← one spatial dimension)

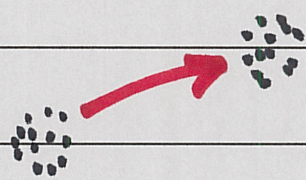


0 $\quad x \quad$ L



molecules vibrating faster

← slower



heat transfer
(convection)

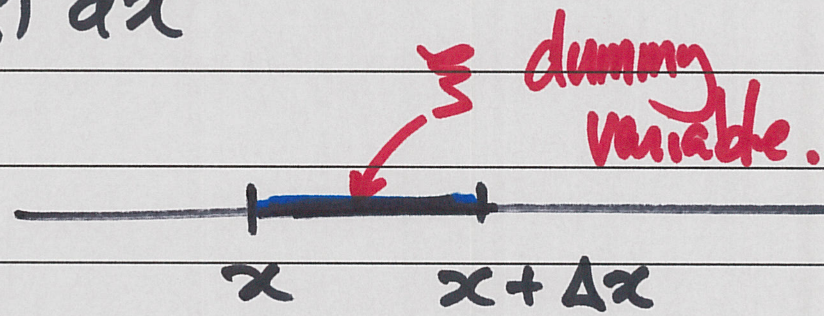
Θ thermal energy density.

$\Theta = \Theta(x, t)$

[redacted]

defining property

total thermal NRG in rod $= \int_0^L \theta(x,t)\,dx$



$$\int_x^{x+\Delta x} \Theta(\cancel{x}\,\xi, t)\, d\cancel{x}\,\xi$$

$\underbrace{\text{Function of time}}$

$$\frac{d}{dt}\int_x^{x+\Delta x} \Theta(\xi,t)\,d\xi$$

$\boxed{\text{rate at which thermal energy enters the region/interval.}}$

[redacted]

physical dimensions of θ ?

<u>Hint</u>: The physical dimensions of velocity <u>are/is</u>

$$\frac{L}{T} = [\theta]$$

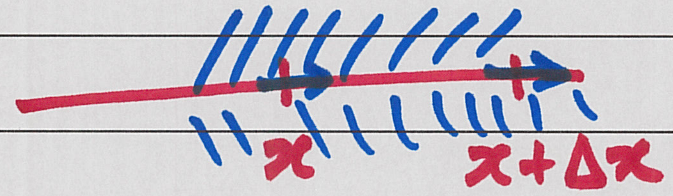
$$[\text{energy}] = [\text{force}] \cdot L$$

$$= \frac{M \cdot L}{T^2} \cdot L$$

$$[\theta] = \frac{[\text{energy}]}{L}$$

[redacted]

(lineal) heat flux $\phi = \phi(x,t)$



ϕ = rate at which thermal energy moves to the right at x.

$$\boxed{\frac{d}{dt}\int_{x}^{x+\Delta x} \theta(\xi,t)\,d\xi = \phi(x,t) - \phi(x+\Delta x,t)}$$

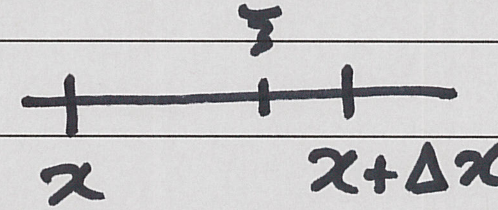
note Δx is positive here.

What can we get from this.

[redacted]

$$\boxed{\frac{d}{dt}\int_x^{x+\Delta x} \theta(\xi,t)\,d\xi = -\left[\phi(x+\Delta x,t)-\phi(x,t)\right]}$$

" ξ = xi "



Differentiate under the integral

$$\int_x^{x+\Delta x} \frac{\partial \theta}{\partial t}(\xi,t)\,d\xi = -\int_x^{x+\Delta x} \frac{\partial \phi}{\partial x}(\xi,t)\,d\xi$$

$$\rightarrow \boxed{\int_x^{x+\Delta x} \left[\frac{\partial \theta}{\partial \cancel{x}\,t} + \frac{\partial \phi}{\partial x}\right] d\xi = 0}$$

Integral form (preliminary) of heat eqn. [redacted]

$$\int_x^{x+\Delta x} \left[\frac{\partial \theta}{\partial t} + \frac{\partial \phi}{\partial x} \right] d\xi = 0$$

for all intervals $[x, x+\Delta x]$ in $[0, L]$.

Fundamental Lemma of Vanishing Integrals.

If f is a continuous function with

$$\int_R f = 0 \text{ for all regions } R,$$

then $f(x) = 0$ for all x.

pointwise form $\quad \dfrac{\partial \theta}{\partial t} = -\dfrac{\partial \phi}{\partial x}$

More general:

External Sources/sinks of thermal energy.

Q defined by

$$\boxed{\int_x^{x+\Delta x} Q \, d\xi} = \text{rate of "other" heat energy change in the interval.}$$

$$[Q] = \frac{[\text{energy}]}{L \cdot T}$$

integral form: $\int \left[\frac{\partial \theta}{\partial t} + \frac{\partial \phi}{\partial x} - Q \right] = 0$

Pointwise form $\frac{\partial \theta}{\partial t} = - \frac{\partial \phi}{\partial x} + Q$

[redacted]

<u>Last function</u> u = <u>temperature</u>

(Θ, ϕ, Q) (easy to measure)

- Law of specific heat $\Theta = c \rho u$

specific heat capacity

mass density

$[\rho] = \frac{M}{L}$

- Fourier's Law

thermal conductivity

$$\boxed{\phi = -K u_x}$$

temperature gradient.

$[u] = [\underline{\underline{temp}}]$

[redacted]

conservation/integral form

$$\int \left[\frac{\partial}{\partial t}(c\rho u) - \frac{\partial}{\partial x}\left(K \frac{\partial u}{\partial x}\right) - Q \right] = 0$$

Pointwise

$$\boxed{\frac{\partial}{\partial t}(c\rho u) = \frac{\partial}{\partial x}\left(K \frac{\partial u}{\partial x}\right) + Q}$$

Other Forms: c, ρ, K constant. f

$$u_t = \underbrace{\frac{K}{c\rho}}_{k \text{ diffusivity.}} u_{xx} + \frac{Q}{c\rho}$$

[redacted]

$$\boxed{u_t = k\,u_{xx} + f}$$

$$Lu = u_t - k\,u_{xx}$$ (linear partial differential operator)

heat/diffusion operator

diffusivity.

$$Lu = f$$

forcing
=
inhomogeneity.

[redacted]

(forms cont.)

homogeneous, constant thermal properties

$$\boxed{u_t = k\, u_{xx}}$$

$\underline{k=1}$ $\boxed{u_t = u_{xx}}$ Parabolic PDE.

$$\Delta u = \sum_{j=1}^{n} \frac{\partial^2 u}{\partial x_j^2} \underset{\underset{n=1}{\uparrow}}{=} \frac{\partial^2 u}{\partial x^2} = u_{xx}$$

[redacted]