Atlanta Lecture Series in Combinatorics and Graph Theory

Abstracts

Increasing paths in countable hypergraphs

Brad Elliot, Emory University

In a classical problem, the vertices or edges of a finite graph are given an ordering, and we are asked how long a path exists in the graph whose sequence of vertices or edges are increasing in order. Under the worst-case ordering, how long of an increasing path can we always find in a given graph? This problem has been well-studied, and has also been extended to graphs with countably many vertices, for which one asks whether there is an infinitely long increasing path under every vertex or edge labeling of the graph. We have now considered this problem for hypergraphs and for other methods of labeling vertices or edges. This presentation provides a mix of results and methods surrounding this problem, and some open questions as well. In particular, we show that any ordering of the vertices of a hypergraph admits an infinite increasing path if and only if the hypergraph contains a subhypergraph in which all degrees are infinite.

This is joint work with Andrii Arman and Vojtech Rodl.

Circular coloring of planar graphs

Dan Cranston, Virginia Commonwealth University

For integers $a \geq 2b > 0$, a circular a/b-flow takes values from $\{\pm b, \pm(b+1), \ldots, \pm(a-b)\}$. We prove that (i) every 10-edge-connected planar graph admits a circular 5/2-flow and (ii) every 16-edge-connected planar graph admits a circular 7/3-flow. Statement (i) was previously proved by Dvořák and Postle, but our proof is much shorter and avoids using computers for case-checking. Further, it has new implications for antisymmetric flows. Statement (ii) is especially interesting because of known 12-edge-connected nonplanar graphs that admit no circular 7/3-flow. Thus, the planarity hypothesis of (ii) is essential. This work is joint with Jiaao Li, Nankai University, China.

The even map color theorem

Mark Ellingham, Vanderbilt University

Most people are familiar with the Four Color Theorem, and many people have heard of the Map Color Theorem, which generalizes the Four Color Theorem to arbitrary surfaces by bounding the chromatic number of a graph embeddable in a surface. In 1976 Joan Hutchinson showed that the bound of the Map Color Theorem can be significantly improved for graphs embeddable with all faces having even degree. Recently we completed Hutchinson's work to prove the Even Map Color Theorem, which shows that Hutchinson's bound is sharp in most cases. The sharpness examples are found using quadrangular or nearly quadrangular embeddings of complete graphs, and we discuss several techniques that can be used to construct these embeddings.

This is joint work with Beifang Chen, Nora Hartsfield, Serge Lawrencenko, Wenzhong Liu, Hui Yang, Dong Ye and Xiaoya Zha.

Decomposing graphs into edges and triangles

Bernard Lidicky, University of Iowa

We prove the following 30-year old conjecture of Gyori and Tuza: the edges of every n-vertex graph G can be decomposed into complete graphs $C_1, \ldots, C_\ell$ of orders two and three such that $|C_1| + \cdots + |C_\ell| \leq (1/2 + o(1))n^2$. This result implies the asymptotic version of the old result of Erdos, Goodman and Posa that asserts the existence of such a decomposition with $\ell \leq n^2/4$. We also discuss removing $o(1)$ term sharpening the result and possible extensions.

The talk is based on joint works with Blumenthal, Kral, Martins, Pehova, Pikhurko, Pfender, Volec.

Graphs with large chromatic number

Alex Scott, Oxford University

If a graph G has large chromatic number, then what can we say about its induced subgraphs? In particular, if G does not contain a large clique, then what else can we guarantee? Thirty years ago, Andras Gyarfas made a sequence of beautiful conjectures on this topic. We will discuss the recent resolution of several of these conjectures, and other related results. Joint work with Maria Chudnovsky, Paul Seymour and Sophie Spirkl.

Graphs with forbidden induced subgraphs

Alex Scott, Oxford University

It is well known that a graph on n vertices need not have complete subgraphs or independent sets of size more than about log n. But what if we consider graphs which do not contain some specific induced subgraph? Erdos and Hajnal conjectured in the 1980s that for every graph H there is a constant c such that every graph on n vertices without an induced copy of H contains a clique or stable set of size n^c. The Erdos-Hajnal conjecture is still very much open, but we will discuss some recent progress and related results. Joint work with Maria Chudnovsky, Paul Seymour and Sophie Spirkl.

Coloring graphs with forbidden minors

Zixia Song, University of Central Florida

Hadwiger's conjecture from 1943 states that for every integer $t \geq 1$, every graph either can be t-colored or has a subgraph that can be contracted to the complete graph on $t + 1$ vertices. Proving that graphs with no K_7 minor are 6-colorable is the first case of Hadwiger's conjecture that is still open. It is not known yet whether graphs with no K_7 minor are 7-colorable. In this talk, I will present our recent results related to Hadwiger's conjecture. Using a Kempe-chain argument along with the fact that an induced path on three vertices is dominating in a graph with independence number two, we prove that every graph with no K_t minor is $(2t-6)$-colorable, where $t \in \{7, 8, 9\}$. We then prove that graphs with no K_8^- minor are 9-colorable, and graphs with no $K_8^=$ minor are 8-colorable. Finally we prove that if Mader's bound for the extremal function for K_t minors is true, then every graph with no K_t minor is $(2t-6)$-colorable for all $t \geq 6$. We believe that the Kempe-chain method we have developed in proving these results is of independent interest.

This is joint work with Martin Rolek.

Infinite random geometric graphs on a circle

Yinon Spinka, University of British Columbia

Abstract: Let (X, d) be a metric space and let V be a dense countable subset of X. Construct a random graph G on V by placing an edge between any two points in V with probability q if the distance between them is less than one (and do so independently for different pairs of points). We are interested in the almost-sure properties of G, or more specifically, of the isomorphism class of G. Such properties may be very sensitive to the metric space (though usually less to V and q). For example, Bonata and Janssen, who initiated the study of these graphs, showed that in the case of the (R^d, L_∞), two independent samples of G are almost surely isomorphic, whereas in the case of (R^d, L_p) with $1 < p < \infty$ (including the Euclidean case $p = 2$), two such samples are almost surely non-isomorphic. We consider the case of a circle R/LZ of length L with its intrinsic metric, and show a surprising dependence of behavior on L.

Joint work with Omer Angel.

The extremal functions for triangle-free graphs with excluded minors

Youngho Yoo, Georgia Institute of Technology

Linklessly embeddable graphs are 3-dimensional analogues of planar graphs which include apex planar graphs. While there is no known analogue of Euler's formula for linkless embeddings, a tight bound of $4n-10$ on the number of edges in linklessly embeddable graphs can be obtained from their excluded minor characterization and a theorem of Mader on the extremal functions for graphs with no K_p minor for small p. We prove an analogue of Mader's theorem for triangle-free graphs, and also show that apex planar graphs satisfy the edge bound of $3n - 9 + \frac{t}{3}$, where t is the number of triangles. This bound is conjectured to hold for all linklessly embeddable graphs. Joint work with Robin Thomas.

On the local version of Sidorenko's conjecture

Fan Wei, Stanford University

A famous conjecture of Erdős-Simonovits and Sidorenko states that if H is a bipartite graph, then the random graph with edge density p has in expectation asymptotically the minimum number of copies of H over all graphs of the same number of vertices and edge density. Lovász proved a local version of this conjecture. Here we extend this result, proving that the local version holds for a graph if and only if the graph is a forest or it has even girth. We will also discuss some results for the local approach for Ramsey multiplicity problem, which is still widely open.