CS 1050 - Proofs
Homework 1
Assigned August 19
Due Thursday, August 26

Determine the following sets; i.e., list their elements if they are not empty or write "empty" if they are empty.

1. $\{n \in \mathbb{N} \text{ s.t. } n^2 = 9\}$.
2. $\{n \in \mathbb{Z} \text{ s.t. } |n| < 7\}$ (where $|-3| = |3| = 3$ is the absolute value).
3. $\{n \in \mathbb{Z}^+ \text{ s.t. } n \text{ is prime and } n \leq 15\}$ (Recall that 1 is not prime!)
4. $\{x \in \mathbb{Q} \text{ s.t. } x^2 = 3\}$

2. a) Prove the following lemma:

Lemma 1 *The sum of two even numbers is even.*

b) Prove the following lemma:

Lemma 2 *The sum of two odd numbers is even.*

c) A multiple of 3 is an integer n that can be written as $n = 3x$ for some integer x. Prove the following lemma:

Lemma 3 *The sum of two integers which are multiples of 3 is also a multiple of 3.*

(Notice that we cannot conclude that the sum of two numbers that are not multiples of 3 is always a multiple of 3 or that it is always not a multiple of 3!! For example, the sum of 1 and 2 *is* a multiple of 3, but the sum of 2 and 2 *is not* a multiple of 3.)

3. a) Prove the following lemma:

Lemma 4 *Let a be an integer such that $a = 3k+1$ where k is an integer. Then the remainder when a^2 is divided by 3 is 1.*

b) Prove the following lemma:

Lemma 5 *Let a be an integer such that $a = 3k+2$ where k is an integer. Then the remainder when a^2 is divided by 3 is 1. (Notice that it is not 2.)*

c) Prove the following lemma:

Lemma 6 *If a is an integer and a^2 is a multiple of 3, then a is also a multiple of 3.*

Warning: It is easy to prove that if a is an integer which is a multiple of 3, then also a^2 is a multiple of 3. *This is not what you are being asked to prove.* Do not confuse the two different statements!

d) Give some examples of the first two lemmas. In other words, just exhibit, say, three integers a satisfying the hypotheses of each lemma and verify that the lemma is true.