CS 1050 –Proofs
Practice Midterm 2

1. Negate the following sentences:

   a) For all integers $n \geq 4$, there exists $c \in \mathbb{R}$ such that $n^{100} \geq 2^n$.

   b) The square of an integer is never odd.

   c) $\forall x \in \mathbb{R}\ \forall y \in \mathbb{R}\ [x > y]$ or $[x < y]$.

   d) $\forall x \in \mathbb{P}\ \exists y \in \mathbb{P}\ [x^2 = y^2]$ and $[x \neq y]$.

2. Prove the following theorem using induction.

Theorem 1. For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be defined by

$$f(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2)$$

for all reals x_1, x_2.

Prove that f is invertible.

4. Let $a_1 = 5$, $a_2 = 13$, and, for $n \geq 2$, let $a_{n+1} = 5a_n - 6a_{n-1}$.

Prove the following theorem.

Theorem 2. For all $n \in \mathbb{Z}^+$, $a_n = 3^n + 2^n$.

5. Prove that for all integers $n \geq 1$, $8^n - 2^n$ is a multiple of 6.