Some statistics of a random graph in the Bollobás–Riordan model

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We consider a random graph G_m^n which was suggested by Barabási and Albert and defined more precisely by Bollobás and Riordan (see [1]). This graph is a model of real-world networks. We deal with some statistics of G_m^n.

In [1] Bollobás and Riordan proved that in this model the degree sequence follows a power law distribution. We study second degrees of vertices in G_m^n. By the second degree of t we mean the number of edges adjacent to the neighbors of t except for the edges adjacent to the vertex t. We show that the distribution of the second degrees in G_1^n is of a power law type as well.

Similarly we can define the k-th degree of a vertex (a quantity $d_k(t)$). For $k \geq 2$, we have calculated the expectation of $d_k(t)$.

In [1] Bollobás and Riordan obtained an asymptotic formula for the diameter of G_m^n. We define a generalisation of this notion — the r-diameter of G_m^n:

$$\mathrm{diam}(G_m^n) = \max_{\substack{i,j \in V \\ i \neq j}} \rho(i,j).$$

Here $\rho(i,j)$ is the distance between i and j in G. We obtain an asymptotic formula for the r-diameters.

References

[1] B. Bollobás, O. M. Riordan, *The degree sequence of a scale-free random graph process*, Random Structures and Algorithms, 18:3 (2001), 279-290.

[2] B. Bollobás, O.M. Riordan, *The diameter of a scale-free random graph*, Combinatorica, 24:1 (2004), 5-34.