A Classification of Tridiagonal Pairs

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Let F denote a field and let V denote a vector space over F with finite positive dimension. We consider a pair of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy the following conditions:

(i) each of A, A^* is diagonalizable;

(ii) there exists an ordering $\{V_i\}_{i=0}^{d}$ of the eigenspaces of A such that $A^* V_i \subseteq V_{i-1} + V_i + V_{i+1}$ for $0 \leq i \leq d$, where $V_{-1} = 0$ and $V_{d+1} = 0$;

(iii) there exists an ordering $\{V_i^*\}_{i=0}^{\delta}$ of the eigenspaces of A^* such that $AV_i^* \subseteq V_{i-1}^* + V_i^* + V_{i+1}^*$ for $0 \leq i \leq \delta$, where $V_{-1}^* = 0$ and $V_{\delta+1}^* = 0$;

(iv) there is no subspace W of V such that $AW \subseteq W$, $A^*W \subseteq W$, $W \neq 0$, $W \neq V$.

We call such a pair a *tridiagonal pair* on V.

We classify up to isomorphism the tridiagonal pairs over an algebraically closed field. To motivate our results we discuss how tridiagonal pairs are related to the orthogonal polynomials from the terminating branch of the Askey-scheme.

This is joint work with Tatsuro Ito and Kazumasa Nomura.