Math 1553 Worksheet §3.1, 3.2

1. Is it possible to write

$$b = \begin{pmatrix} -3 \\ -9 \\ 7 \end{pmatrix} \text{ as a linear combination of } \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \text{ and } \begin{pmatrix} -1 \\ -5 \\ -6 \end{pmatrix}?$$

   If your answer is no, justify why not. If your answer is yes, write b as a linear combination of those four vectors.

2. Let

$$A = \begin{pmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{pmatrix}, \qquad b = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

   Is b in the span of the columns of A? Justify your answer.

3. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.

   a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.

   b) The span of any set contains the zero vector.

4. Zander has challenged you to find his hidden treasure, located at some point (a, b, c). He has honestly guaranteed you that the treasure can be found by starting at the origin and taking steps in directions given by

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} \qquad v_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}.$$

   By decoding Zander's message, you have discovered that the treasure's first and second entries are (in order) -4 and 3.

   a) What is the treasure's full location?

   b) Give instructions for how to find the treasure by only moving in the directions given by v_1, v_2, and v_3.