Math 1553 Worksheet §4.1, 4.3

Linear Transformations

1. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

   a) Counterclockwise rotation by $32°$ in $\mathbf{R}^2$.

   b) The transformation $T: \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

   c) The transformation $T: \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

   d) The matrix transformation with matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.

   e) The matrix transformation with matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

2. Say A is an $m \times 2$ matrix. Let $T: \mathbf{R}^2 \to \mathbf{R}^m$ be the transformation defined by $T(x) = Ax$. If the columns of A are linearly independent, what does the image of T look like geometrically? What if they're linearly dependent?

3. For each matrix A, describe what the associated matrix transformation T does to $\mathbf{R}^3$ geometrically.

a) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ **c)** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

4. The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(1,1,1)$. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of $45°$ in a counterclockwise direction about the z-axis, and then projected onto the xy-plane. Find the matrix for this transformation.