

SOLUTIONS 3:00

① a) F F T T F

b) 0, *, *

c) B, C

d) I, P, P



③ X
✓
X

④ 8-3=5
M
4, 7
0

⑤ a) $B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ b) not possible c) $\begin{pmatrix} 3 & -1 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$ e) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

⑥ $\sim \begin{pmatrix} 0 & -k & -3k \\ 0 & 1 & 3k \\ 1 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & -3k+3k^2 \end{pmatrix} \Rightarrow k=0, 1$

⑦ easy way: use columns of A, $\vec{b} = x_1 \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

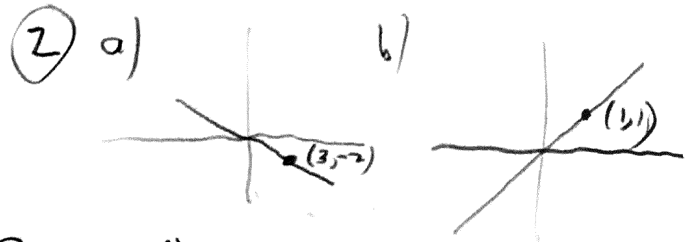
standard method: $\left(\begin{array}{cc|c} 1 & 0 & b_1 \\ -4 & -1 & b_2 \\ 5 & 1 & b_3 \end{array} \right) \sim \left(\begin{array}{cc|c} \cdot & \cdot & \cdot \\ 0 & -1 & b_2 + 4b_1 \\ 0 & 1 & b_3 - 5b_1 \end{array} \right)$

$\Rightarrow b_3 - b_1 + b_2 = 0$, or $\vec{b} = b_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

	1	2	3	4	5	6	7	TOTAL
Q								
P	15	2	3	5	10	7	8	50

SOLUTIONS: 4:30

- ① a) false, true, true, false, false
 b) $1 \neq$, not possible, \neq not zero
 c) A, B
 d) a, c only



③ 1st, 3rd defined

- ④ a) 2 b) N c) $k=0$ d) 4, 7

- ⑤ a) Not possible, b) ^{only answer} $\begin{pmatrix} 0 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ e) ^{unique answer} $\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$

⑥ $\begin{pmatrix} k & 0 & -k \\ 0 & 1 & 3k \\ -1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 0 & -k & -2k \\ 0 & 1 & 3k \\ -1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & -2k+3k^2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \Rightarrow k=0, 2/3$

⑦ easy way: columns of A, $\left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} \right\}$, so $\vec{x} = c_1 \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + c_2 \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

longer: $\begin{pmatrix} 1 & 0 & | & b_1 \\ 3 & -1 & | & b_2 \\ -4 & 1 & | & b_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & b_1 \\ 0 & -1 & | & b_2 - 3b_1 \\ 0 & 1 & | & b_3 + 4b_1 \end{pmatrix} \sim \begin{pmatrix} \cdot & \cdot & | & \cdot \\ \cdot & \cdot & | & \cdot \\ 0 & 0 & | & b_3 + b_1 + b_2 \end{pmatrix}$

\Rightarrow need $b_1 + b_2 + b_3 = 0$

$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} -b_2 - b_3 \\ b_2 \\ b_3 \end{pmatrix} = b_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + b_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

QUESTION	1	2	3	4	5	6	7	TOTAL
POINTS	15	2	3	5	10	7	8	50

SOLUTIONS 6:00

① a) f, t, t, t, t

b) I, I, P

c) 2, *, 2, *

② •
x
x
•
x

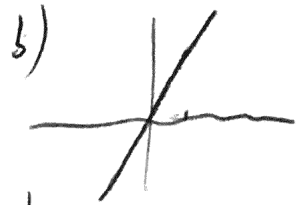
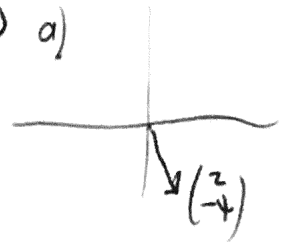
③ $\mathbb{Q}, \mathbb{R}^3, \mathbb{R}^2$

b) $\begin{pmatrix} 2 & 4 & 0 \\ 3 & 6 & -7 \end{pmatrix}$

c) Y

d) N

④ a)



⑤ a) $\begin{pmatrix} 0 & 0 \\ 0 & * \end{pmatrix}$ b) not pass. c) not pass. d) $\begin{pmatrix} 1 & 6 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 1 & | & 2 \\ -1 & 1 & | & 3 \end{pmatrix}$

⑥ $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$

⑦ $\sim \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\vec{X} = \begin{pmatrix} 1-x_2-x_3 \\ x_2 \\ x_3 \\ 2-x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

⑧ $\begin{pmatrix} 1 \\ 0 \\ t \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -t \end{pmatrix}, \begin{pmatrix} 1 \\ t \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & t \\ 0 & -t & -2+t \end{pmatrix} \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & 0 & * \end{pmatrix}$

$* = t^2 + t - 2 \Rightarrow (t+2)(t-1) = 0 \Rightarrow t = -2, +1$

Q	1	2	3	4	5	6	7	8	TOTAL
POINTS	11	5	5	2	10	2	8	7	50