Worksheet 2b: Separable diff eqns

1. Verify that $y = e^{-x}\tan(2e^x)$ is a solution to the differential equation $y' + y = \frac{2}{1+4e^{2x}}$ satisfying the initial condition $y(-\ln 2) = \frac{\pi}{2}$.

2. Solve the differential equations. Check your answers.

(a) $\frac{dy}{dx} = 3x^2e^{-y}$

(b) $\sqrt{x}\frac{dy}{dx} = e^{y+\sqrt{x}}$, $x > 0$

(c) $(\sec x)\frac{dy}{dx} = e^{y+\sin x}$

(d) $\frac{dy}{dx} = xy + 3x - 2y - 6$

3. For the chance of δ-glucono lactone into gluconic acid, the rate at which the amount of the former substance changes into the latter is $\frac{dy}{dt} = -0.6y$, when t is measured in hours. If there are 100 grams of δ-glucono present at time $t = 0$, how many grams are present after the first hour? Also, how long does it take for there to be 50 grams remaining?

4. The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium.

5. Newton's Law of Cooling states that the rate of change of the temperature T of an object is proportional to $T - T_A$ where T_A is the ambient room temperature. Suppose that a cup of soup cooled from $90\,^\circ\mathrm{C}$ to $60\,^\circ\mathrm{C}$ after 10 min in a room whose temperature was $20\,^\circ\mathrm{C}$. How much longer would it take the soup to cool to $35\,^\circ\mathrm{C}$? At what point does the soup temperature match the ambient room temperature *exactly* according to Newton's Law of Cooling?