Practice Exam 1

1. Find the domain and range of the functions $f(x) = |x| - 2$, $g(x) = 3^{2-x} + 1$, $h(x) = \frac{3-x}{\sqrt{25-x^2}}$.

2. Find the domain and range of the function

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } -4 \leq x \leq 0 \\ \sqrt{x} & \text{if } 0 < x \leq 4 \end{cases}$$

3. Find $f \circ g$, $g \circ f$, $(f \circ g)(-1)$ and $(g \circ f)(2)$ if $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{\sqrt{x+2}}$.

4. Sketch the graph of

$$f(x) = \begin{cases} -x - 2 & \text{if } -4 \leq x \leq -1 \\ -1 & \text{if } -1 < x \leq 1 \\ x - 2 & \text{if } 1 < x \leq 2 \end{cases}$$

For what values of x is the function $f(x)$ continuous? For what values does the function have a one-sided limit but NOT a two-sided limit?

5. Find the domain of $f(x) = 1 + e^{-\sin(x)}$.

6. Find the largest $\delta > 0$ such that if $|x - 23| < \delta$ then $|f(x) - 4| < \varepsilon$ for $\varepsilon = 1$, where $f(x) = \sqrt{x - 7}$.

7. At what points is the function $f(x) = \frac{x \tan x}{x^2+1}$ continuous? Repeat question for $g(x) = \sqrt{3x - 1}$.

8. Find the limits.

(a) $\displaystyle\lim_{h \to 0^+} \frac{(x+h)^2 - x^2}{h}$

(b) $\displaystyle\lim_{x \to \pi^-} \csc(x)$

(c) $\displaystyle\lim_{x \to 0} \frac{8x}{3\sin x - x}$

(d) $\displaystyle\lim_{x \to \infty} \frac{x^4 + x^3}{12x^3 + 128}$

(e) $\displaystyle\lim_{x \to \infty} \frac{\cos x - 1}{x}$

9. Suppose $\lim_{x \to a} f(x) = -7$ and $\lim_{x \to a} g(x) = 0$. Find the following limits

(a) $\displaystyle\lim_{x \to a} 3f(x) - g(x)$

(b) $\displaystyle\lim_{x \to a} \frac{f(x)}{7 - g(x)}$

(c) $\displaystyle\lim_{x \to a} f(x) \cdot g(x)$

10. Suppose $\lim_{x \to 0} \left(\frac{4 - f(x)}{x} \right) = 1$. Find $\lim_{x \to 0} f(x)$.