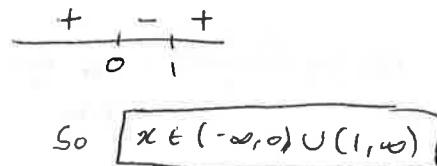


Practice Exam 1

1. Find the domain of $\frac{x}{\sqrt{x^2 - x}}$. Express your answer in interval notation.

need $\sqrt{x^2 - x} \neq 0 \quad \left. \begin{array}{l} x^2 - x > 0 \\ x(x-1) > 0 \end{array} \right\}$
 and $x^2 - x \geq 0$



2. Is the function $f(x)$ one-to-one?

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$$

No. $f(-1) = f(0)$.

3. Write the domain and range of $f(x) = |x + 2| - 1$ in interval notation.

D: $(-\infty, \infty)$
R: $[-1, \infty)$

4. What is the average rate of change of $f(x) = \log_2(x + 3)$ on the interval $[1, 5]$.

$$\frac{\log_2(5+3) - \log_2(1+3)}{5-1} = \frac{\log_2 8 - \log_2 4}{4} = \frac{3-2}{4} = \boxed{1/4}$$

5. Compute the limits.

$$(i) \lim_{x \rightarrow 0^+} \frac{x}{|x|} = \boxed{1}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x} = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{x+1 + x-1}{(x-1)(x+1)} = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{2x}{(x-1)(x+1)} = \lim_{x \rightarrow 0} \frac{2}{x^2-1} = \boxed{-2}$$

$$(iii) \lim_{x \rightarrow \infty} \frac{2x^3}{4x^3+x^2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

$$(iv) \lim_{x \rightarrow \infty} e^{-x} = \boxed{0}$$

$$(v) \lim_{x \rightarrow -2} \frac{x}{(x+2)^2} = \boxed{-\infty \text{ DNE}}$$

6. Let $f(x) = 2x + b$, where b is a constant, and note $\lim_{x \rightarrow 2} f(x) = 4 + b$. Find the largest $\delta > 0$ such that, for $\varepsilon = 2$,

$$|x - 2| < \delta \implies |f(x) - (4 + b)| < \varepsilon.$$

$$\begin{aligned} |2x+b - 4-b| &< 2 &\Leftrightarrow |x-2| &< 1 \\ \Leftrightarrow -2 &< 2x - 4 < 2 \\ \Leftrightarrow 2 &< 2x < 6 \\ \Leftrightarrow 1 &< x < 3 \end{aligned}$$

$$\boxed{\delta = 1}$$

7. For what values of a is $f(x)$ continuous for all real x .

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ 2x + a & \text{if } x \geq 1 \end{cases}$$

$$\text{Need } \lim_{x \rightarrow 1^-} f(x) = f(1)$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 1 = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} 0 = 2 + a \Rightarrow \boxed{a = -2}$$

$$f(1) = 2(1) + a = 2 + a$$

8. Find derivative of $f(x) = \frac{1}{\sqrt{x}}$ at $x = 1$ using the definition of the derivative.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h} \cdot \sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{x - x - h}{\sqrt{x+h} \sqrt{x} (\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{-1}{h \sqrt{x+h} \sqrt{x} (\sqrt{x+h} + \sqrt{x})} = \boxed{f'(1) = -\frac{1}{2}} \end{aligned}$$

9. Suppose $f(1) = 2$ and $f'(1) = 3$, for a function $f(x)$ which is differentiable at $x = 1$. Find the equation of the line tangent to the graph $y = f(x)$ at $x = 1$.

$$\begin{aligned} y &= mx + b \\ 2 &= 3(1) + b \end{aligned}$$

$$\boxed{y = 3x - 1}$$

$$b = -1$$

