Math 1501 **Exam 1** Fall 2013

Instructor: Sal Barone

Name: KEY

GT username: ____________________

Circle your TA/section: (N1) Daniel (N2) Rebecca (C1) Rachel (C2) Lily

* Red indicates "important bits"

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work to receive full credit.
4. Write your answers in the box provided.
5. Good luck!

Page	Max. Possible	Points
1	20	
2	18	
3	20	
4	22	
5	20	
Total	100	

1. Find the domain of $\dfrac{\ln(x-1)}{\sqrt{25-x^2}}$. Express your answer in interval notation.(10 pts.)

need $x - 1 > 0$ $\quad \boxed{x > 1}$

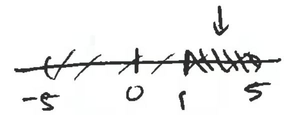
and $25 - x^2 \geq 0$

and $\sqrt{25-x^2} \neq 0$

$\left.\right\}$ $25 - x^2 > 0$

$x^2 < 25$

$\boxed{|x| < 5}$



$x \in (1, 5)$

$\boxed{(1,5)}$

2. For what x-values is the function $f(x)$ NOT continuous? (10 pts.)

$$f(x) = \begin{cases} |x| & \text{if } \; x \leq -1 \\ 2x+1 & \text{if } \; -1 < x < 2 \\ \frac{x-7}{x-3} & \text{if } \; x \geq 2 \end{cases}$$

$\lim\limits_{x \to -1^+} f(x) = 2(-1)+1 = -1 \overset{?}{=} f(-1) = |-1| = 1$ ✗ $\boxed{\text{not cont. @ } x=-1}$

$\lim\limits_{x \to 2^-} f(x) = 2(2)+1 = 5 \overset{?}{=} f(2) = \frac{2-7}{2-3} = \frac{-5}{-1} = 5$ ✓ $\boxed{\text{cont @ } x=2}$

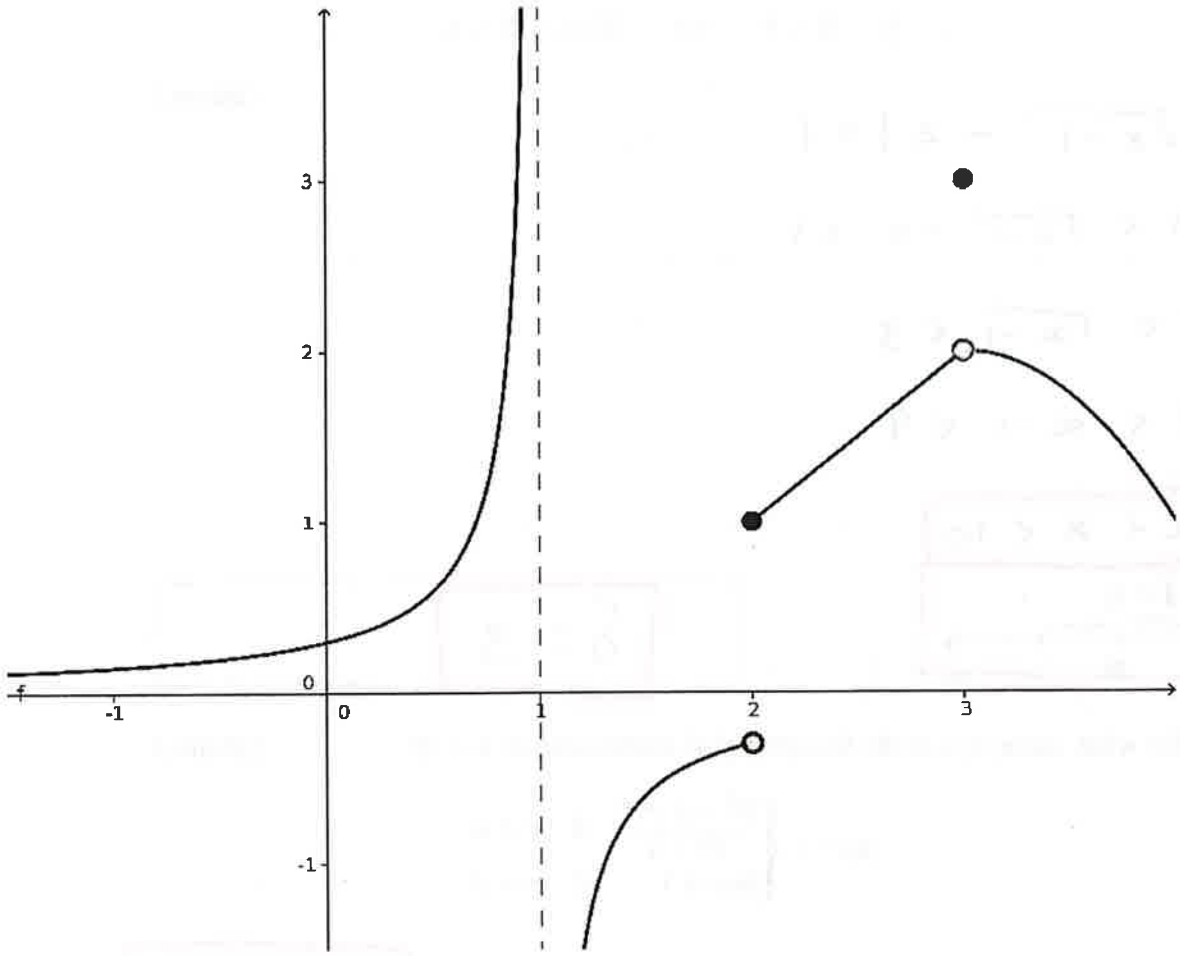
$|x|$ cont on $(-\infty, -1]$

$2x+1$ cont on $(-1, 2)$

$\frac{x-7}{x-3}$ has $\boxed{\text{discontinuity at } x=3}$ in $[2, \infty)$.

$\boxed{x = -1, 3}$

3. Use the graph to answer the questions. (3 pts. each)



(A) $\lim_{x \to 1^-} f(x) = \boxed{+\infty \text{ DNE}}$

(D) $\lim_{x \to 3} f(x) = \boxed{2}$

(B) $\lim_{x \to 2^+} f(x) = \boxed{1}$

(E) $f(3) = \boxed{3}$

(C) $\lim_{x \to 2} f(x) = \boxed{\text{DNE}}$

(F) On what intervals is $f(x)$ continuous?

$\boxed{(-\infty, 1) \cup (1, 2) \cup (2, 3) \cup (3, \infty)}$

4. Let $f(x) = \sqrt{x-1}$, and note that $\lim_{x \to 5} f(x) = 2$. Find the largest δ for which

$$|x-5| < \delta \quad \Longrightarrow \quad |f(x) - 2| < 1.$$

(10 pts.)

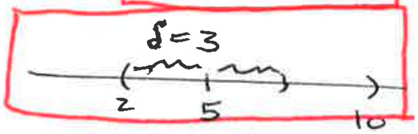
$$\left|\sqrt{x-1} - 2\right| < 1$$

$$\Leftrightarrow \quad -1 < \sqrt{x-1} - 2 < 1$$

$$\Leftrightarrow \quad 1 < \sqrt{x-1} < 3$$

$$\Leftrightarrow \quad 1 < x-1 < 9$$

$$\Leftrightarrow \quad \boxed{2 < x < 10}$$



$$\boxed{\delta = 3}$$

5. For what value of a is the function $g(x)$ continuous at $x = 2$? (10 pts.)

$$g(x) = \begin{cases} \dfrac{x^2 - x - 2}{x^2 - 4} & \text{if } \quad x \neq 2 \\ 3ax + 1 & \text{if } \quad x = 2 \end{cases}$$

need $\lim_{x \to 2} f(x) = f(2)$.

$$\boxed{6a + 1 = \frac{3}{4}}$$

$$6a = -\frac{1}{4}$$

$$\boxed{a = -1/24}$$

* $f(2) = 3a(2) + 1 = 6a + 1$

* $\lim_{x \to 2} f(x) = \lim_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 4} = \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x+2)} = \dfrac{3}{4}$

$$\boxed{a = -1/24}$$

6. Find the derivative $f'(-1)$ using the definition of the derivative, where $f(x) = \frac{2}{x}$. (12 pts.)

$$\boxed{\lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}} = \lim_{h \to 0} \frac{1}{h} \frac{2x - 2(x+h)}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{\cancel{2x} - \cancel{2x} - 2h}{x(x+h)} = \boxed{\lim_{h \to 0} \frac{-2}{x(x+h)}}$$

$$= \frac{-2}{x^2}$$

$$\boxed{-2/x^2}$$

7. Suppose $f(x)$ satisfies $f(4) = 2$ and the derivative of $f(x)$ is $f'(x) = \frac{1}{2\sqrt{x}}$. What is the equation of the line tangent to the curve $y = f(x)$ at $x = 4$? (10 pts.)

$\boxed{y = mx + b}$ $f(4) = 2$ means $x = 4$ and $y = 2$.

$2 = m \cdot 4 + b$ $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$ means $\boxed{m = \frac{1}{4}}$

$2 = \frac{1}{4} \cdot 4 + b = 1 + b$

$\boxed{b = 1}$

$$\boxed{y = \frac{1}{4}x + 1}$$

8. Suppose f, g, h are all functions from $\mathbb{R}$ to $\mathbb{R}$, and that $f(1) = f(2) = 3$, $g(1) = 2$, $g(3) = 4$, $h(2) = 1$, and $h(3) = 5$. Find the following: (2 pts each)

(i) $f \circ h(2) = f(1) = \boxed{3}$

(ii) $g \circ f(1) = g(3) = \boxed{4}$

9. Find the limits. (4 pts each)

(i) $\lim_{x \to 1^+} \dfrac{|1-x|}{x-1} = \lim\limits_{x \to 1^+} \dfrac{-(1-x)}{x-1} = \boxed{1}$

(ii) $\lim_{x \to \infty} \dfrac{x^3 - x^2 - x + 1}{3x^3 - 100} = \boxed{1/3}$

(iii) $\lim_{x \to 3^-} \dfrac{x^2 - 9}{x^2 - 3x} = \lim\limits_{x \to 3^-} \dfrac{(x+3)\cancel{(x-3)}}{x\cancel{(x-3)}} = \dfrac{6}{3} = \boxed{2}$

(iv) $\lim_{t \to \infty} \dfrac{2\sqrt{2}}{3e^{-t} + 2} = \dfrac{2\sqrt{2}}{3 \cdot 0 + 2} = \dfrac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$