Math 1501 **Exam 1** Summer 2015

Instructor: Sal Barone

Name: KEY

GT username: ______________

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please $\boxed{\text{BOX}}$ your answers.
5. Good luck!

Page	Max. Possible	Points
1	30	
2	30	
3	20	
4	20	
Total	100	

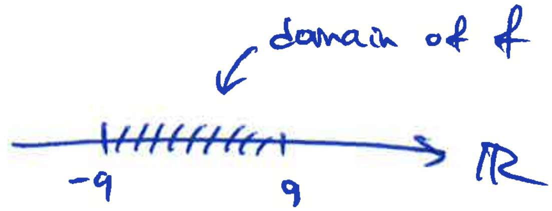
1. Find the domain of the function $f(x) = 3\sqrt{81 - x^2}$. Express your answer in interval notation. (10 pts.)

Require $81 - x^2 \geq 0$

$x^2 \leq 81$

$-9 \leq x \leq 9$

$\boxed{[-9, 9]}$



2. Find the value of a that makes the function continuous at $x = 2$ (20 pts.)

$$f(x) = \begin{cases} -2x + 1 & \text{if } x < -1, \\ x^2 - a & \text{if } -1 \leq x \leq 2, \\ \dfrac{x-2}{x^3 - 2x^2} & \text{if } 2 < x. \end{cases}$$

Require $f(2) = \lim_{x \to 2} f(x)$

$f(2) = 2^2 - a = 4 - a$

and

$\lim_{x \to 2^-} f(x) = f(2)$ already. So,

need

$f(2) = \lim_{x \to 2^+} f(x)$

$\Rightarrow 4 - a = \lim_{x \to 2^+} \dfrac{x-2}{x^3 - 2x^2} = \lim_{x \to 2^+} \dfrac{\cancel{x-2}}{x^2(\cancel{x-2})} = \dfrac{1}{4}$

Solve.

$4 - a = \frac{1}{4}$

$\boxed{a = 15/4}$

3. Find the derivative $f'(x)$ (15 pts.)

QUOTIENT rule $f(x) = \dfrac{2x-1}{x^2+1}$.

$$f'(x) = \frac{(x^2+1)(2) - (2x-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$= \boxed{-2x^2+2x+2 / (x^2+1)^2}$$

4. A particles position in cm after t seconds is given by the function

$$s(t) = \frac{\cos(3t)}{t+1} + 1.$$

Find the velocity of the particle at time $t = 0$. (15 pts.)

$$s'(t) = v(t) = \frac{(t+1)(-\sin(3t)\cdot 3) - \cos(3t)\cdot 1}{(t+1)^2} + 0$$

$$v(0) = \frac{1\cdot 0 - 1}{1^2} = \boxed{-1}$$

5. Find the limits if they exist. Otherwise, write $+\infty$ DNE, $-\infty$ DNE, or just DNE.

(5 pts. each)

(a) $\displaystyle\lim_{x\to\infty} \frac{3x^2-3x+3}{2x^3+10x} = \boxed{0}$

(b) $\displaystyle\lim_{x\to 5^+} \frac{4x-5}{30-6x} = \boxed{-\infty \text{ DNE}}$

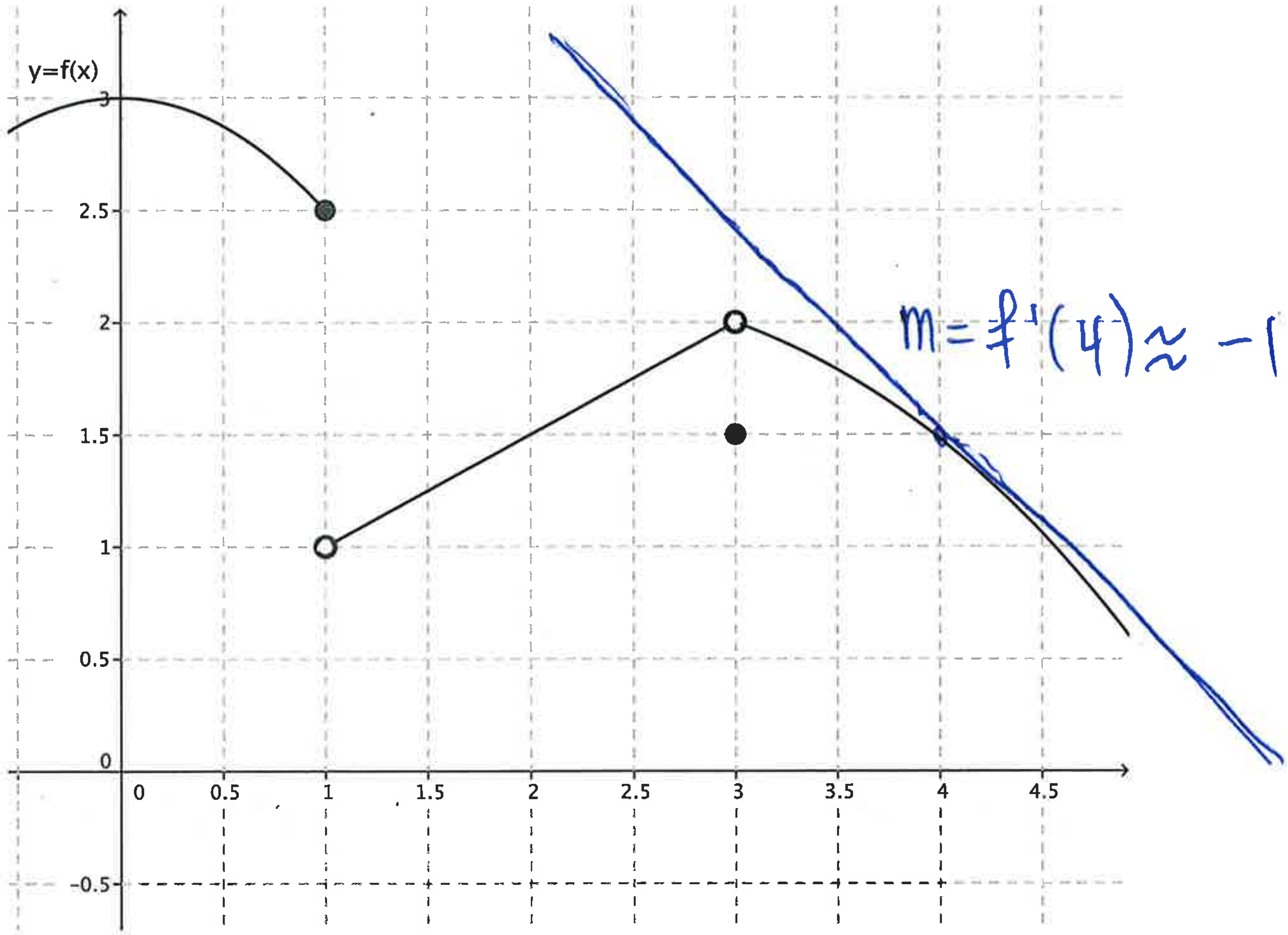
(c) $\displaystyle\lim_{x\to -1} \frac{2x\cos(\pi x)}{x^2+1} = \boxed{1}$

(d) $\displaystyle\lim_{x\to 3} \frac{2x-6}{9-x^2} = \boxed{-1/3}$

b/c $\displaystyle\lim_{x\to 3} \frac{2x-6}{9-x^2} = \lim_{x\to 3} \frac{2\cancel{(x-3)}^{(-1)}}{\cancel{(3-x)}(3+x)} = \frac{-2}{6} = -1/3.$

$\displaystyle\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -1$!!

6. Use the figure below to answer the questions. The figure depicts the graph of the function $y = f(x)$. (4 pts. each)



(a) $\lim_{x \to 1+} f(x) =$ 1

(b) $\lim_{x \to 1} f(x) =$ DNE

(c) $\lim_{x \to 3} f(x) =$ 2

(d) Give an estimate for $f'(4)$ and sketch the tangent line at $x = 4$ in the figure above.

$f'(4) \approx -1$

(e) For what values of x is $f(x)$ continuous? Give your answer in interval notation.

$(-\infty, 1) \cup (1, 3) \cup (3, \infty)$