

MATH 1552· FALL 2015
INTEGRAL CALCULUS
Instructor: Rainer Sinn

Worksheet #15
7 October 2015

1. Evaluate the following integrals using any method we have learned.

- (a) $\int \frac{\sin^3(x)}{\cos(x)} dx$
- (b) $\int \frac{x}{\sqrt{x^2+2x-3}} dx$
- (c) $\int x \arctan(x^2) dx$
- (d) $\int \frac{\cos(x)}{4+\sin^2(x)} dx$
- (e) $\int \cos[\ln(x)] dx$
- (f) $\int \frac{1}{x(x^2+x+1)} dx$

2. Evaluate the following limits:

- (a) $\lim_{x \rightarrow 0} \frac{\ln(1/\cos(x))}{x^2}$
- (b) $\lim_{x \rightarrow \infty} \left(\cos\left(\frac{1}{x}\right)\right)^x$

3. Evaluate the following integrals.

- (a) $\int_0^1 \frac{t+1}{\sqrt{t^2+2}} dt$
- (b) $\int_0^2 \frac{1}{\sqrt{|x-1|}} dx$
- (c) $\int_{-\infty}^{\infty} 2x \exp(-x^2) dx$

4. (a) State the Direct Comparison Test with the necessary assumptions.

- (b) Does the improper integral $\int_1^{\infty} \frac{1}{1+\exp(x)} dx$ exist?
- (c) Does the improper integral $\int_1^{\infty} \frac{\exp(x)}{x} dx$ exist?

5. Find the general solution to the equation:

$$(y \ln x)y' = \frac{y^2 + 1}{x}.$$

6. A radioactive substance loses 20% of its mass each year. Find the half-life (i.e., the time it requires to have half of the substance remaining) of the substance.