KEY

Worksheet

Problem 1 *Evaluate the following integrals using the fundamental theorem of calculus.*

① $\int_0^1 x^{\pi-1}dx$ ② $\int_0^1 \pi^{x-1}dx$ ③ $\int_0^1 (t+1)(t^2+4)dt$

④ $\int_0^1 \cos^2 t dt$ ⑤ $\int_0^1 \frac{dx}{\cos^2 x}$ ⑥ $\int_{\pi/2}^{\pi} \frac{sin(2x)}{2sinx}dx$

⑦ $\int_0^1 \frac{dt}{(t+1)(t+2)}$ ⑧ $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)dx$

① $= \frac{1}{\pi} x^{\pi} \Big|_0^1 = \frac{1}{\pi}$

② $= \int_0^1 e^{(x-1)\ln\pi} dx = \frac{1}{\ln\pi} e^{(x-1)\ln\pi} \Big|_0^1$

$= \frac{1}{\ln\pi}\left(1 - e^{-\ln\pi}\right) = \frac{1}{\ln\pi}\left(1 - \frac{1}{\pi}\right)$

③ $= \int_0^1 t^3 + t^2 + 4t + 1 \, dt = \frac{1}{4} + \frac{1}{3} + 2 + 1 = \frac{43}{12}$

④ $= \int_0^1 \frac{1+\cos 2t}{2} dt = \frac{t}{2} + \frac{\cos 2t}{4} \Big|_0^1 = \frac{1+\cos(2)}{4}$

⑤ $= \tan x \Big|_0^1 = \tan(1)$

⑥ $= \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx = \sin x \Big|_{\frac{\pi}{2}}^{\pi} = -1$

⑦ $= \int_0^1 \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt$

$= \ln \frac{t+1}{t+2} \Big|_0^1$

$= \ln\frac{2}{3} - \ln\frac{1}{2} = \ln\frac{4}{3}$

⑧ $= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos x + \cos x)\, dx$

$+ \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(\cos x - \cos x) dx$

$= \int_0^{\frac{\pi}{2}} \cos x dx = \sin x \Big|_0^{\frac{\pi}{2}} = 1$

Problem 2 *Evaluate dy/dx for the following functions.*

① $y = \int_0^x \sqrt{1+t^2}dt$ ② $y = \int_{\sqrt{x}}^0 \sin(t^2)dt$ ③ $y = \int_2^{x^2} sin(x^2)sin(t^2)dt$

① $y' = \sqrt{1+x^2}$

② $y' = -\frac{d}{dx}\int_0^{\sqrt{x}} \sin(t^2)dt$

$= -\sin x \cdot \frac{d}{dx}\sqrt{x}$

$= -\frac{\sin x}{2\sqrt{x}}$

③ $y = \sin(x^2) \int_2^{x^2} \sin(t^2)\, dt$

$y' = 2x\cos(x^2) \int_2^{x^2} \sin(t^2)\, dt$

$+ 2x \sin(x^2)\sin(x^4)$

Problem 3 *Using u-substitution to evaluate the following indefinite integrals*

① $\displaystyle\int \frac{dx}{e^{-x}+e^{x}}$ ② $\displaystyle\int \frac{dx}{x\ln x}$

③ $\displaystyle\int \sqrt{\sin x}\cos^3 x\,dx$ ④ $\displaystyle\int \frac{dx}{1+e^{x}}$

① $$= \int \frac{e^x\,dx}{1+(e^x)^2} \overset{u=e^x}{=\!=} \int \frac{du}{1+u^2} = \arctan u + C = \arctan(e^x) + C$$

② $$\overset{u=\ln x}{=\!=} \int \frac{du}{u} = \ln u + C = \ln(\ln x) + C$$

③ $$= \int \sqrt{\sin x}\,\cos x \cdot \cos^2 x\,dx = \int \sqrt{\sin x}\,(1-\sin^2 x)\cos x\,dx$$

$$\overset{u=\sin x}{=\!=} \int \sqrt{u}\,(1-u^2)\,du = \int \sqrt{u} - u^{5/2}\,du$$

$$= \frac{2}{3}u^{\frac{3}{2}} - \frac{2}{7}u^{\frac{7}{2}} + C = \frac{2}{3}(\sin x)^{\frac{3}{2}} - \frac{2}{7}(\sin x)^{\frac{7}{2}} + C$$

④ $$\underset{dx=\frac{dt}{t}}{\overset{x=\ln t}{=\!=}} \int \frac{dt}{t(t+1)} = \int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt = \ln\frac{t}{t+1} + C = \ln\frac{e^x}{e^x+1} + C$$

$$= x - \ln(e^x+1) + C$$