Math 1552
Summer 2023
Exam 1 GLC
June 8 @ 5:30pm
Time limit: 75 Minutes

Name (Print): Key

Canvas email: ____________________

Teaching Assistant/Section: ____________________

GT ID:									

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- $\boxed{\textbf{Place a box}}$ around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

GLC

1. (2 points) Suppose that $f(x)$ is a function which is non-negative and not constant on the interval $[3,5]$ and $\int_3^5 f(x)\,dx = A$. Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
●	○	If we estimate A using the lower-sum method with $n = 4$ rectangles, then we will get a value smaller than A.
○	●	If $f(x)$ is increasing on $[3,5]$ and we estimate A using the right-endpoint method with $n = 4$ rectangles, then we will get a value smaller than A.

2. (3 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-5}^5 f(x)\,dx = 8$, $\int_2^5 f(x)\,dx = 1$, and $\int_{-2}^0 g(x)\,dx = 5$, find $A = \int_0^2 2f(x) - g(x)\,dx$.

$A = \boxed{11}$

$$\int_0^2 2f(x)-g(x)\,dx = 2\int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$$

↓ g odd

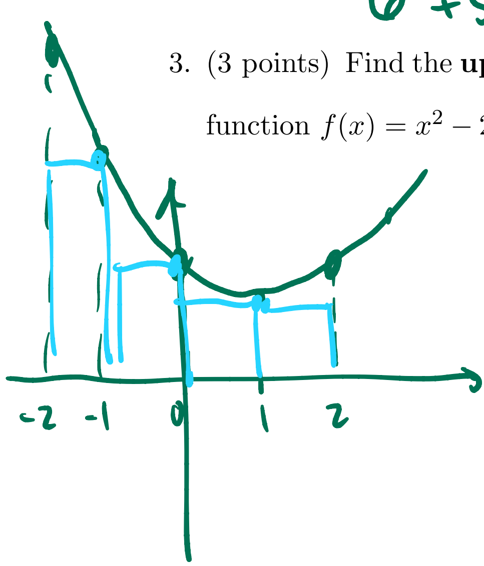
$$= 2\left(\int_0^5 f(x)\,dx - \int_2^5 f(x)\,dx\right) - (-5)$$

↓ f even

$$= 2(4 - 1) + 5$$

$$= 6 + 5$$

3. (3 points) Find the **upper-sum** Riemann sum estimate for the area between the x-axis and the function $f(x) = x^2 - 2x + 2$ over the interval $[-2,2]$ using $n = 4$ rectangles. area $\approx \boxed{19}$



$f(-2) = 4+4+2 = 10$) 10

$f(-1) = 1+2+2 = 5$) 5

$f(0) = 2$

$f(1) = 1$) 2

$f(2) = 2$) 2

$10+5+2+2$

GLC

4. (4 points) Compute $F'(x)$ using the fundamental theorem of calculus.

$$F(x) = \int_2^{\sqrt{x}} t\sqrt{1+t^4}\, dt$$

$G(x) = \int_2^{x} t\sqrt{1+t^4}\, dt$

$\Rightarrow G'(x) = x\sqrt{1+x^4}$

by FTC

$F(x) = G(\sqrt{x})$

$\Rightarrow F'(x) = G'(\sqrt{x}) * (\sqrt{x})'$

$= \sqrt{x}\sqrt{1+(\sqrt{x})^4} \cdot \frac{1}{2\sqrt{x}}$

$= \boxed{\frac{1}{2}\sqrt{1+x^2}}$

5. (4 points) Find the general antiderivative.

$$\int \frac{2}{\sqrt{4-x^2}}\, dx = \int \frac{\cancel{2}\,1}{\cancel{\sqrt{4}}} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\, dx$$

u-sub Box

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$= \int \frac{1}{\sqrt{1-u^2}} \cdot 2\, du$

$= 2\sin^{-1}(u) + C$

$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) + C}$

GLC

6. (24 points) Integrate.

(a) $\int_0^{\sqrt{7}} x(x^2+1)^{1/3}\,dx = \int_*^* u^{1/3} \cdot \frac{1}{2}\,du$

u-sub box

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \left.\frac{\frac{1}{2}u^{4/3}}{4/3}\right|_*^* = \frac{1}{2}\cdot\frac{3}{4}(x^2+1)^{4/3}\Big|_0^{\sqrt{7}}$

$= \frac{3}{8}(7+1)^{4/3} - \frac{3}{8}(0+1)^{4/3} = \frac{3}{8}\cdot 8^{4/3} - \frac{3}{8}$

$= \frac{3}{8}\cdot 16 - \frac{3}{8} = \frac{48}{8} - \frac{3}{8}$

$= \frac{48-3}{8} = \boxed{\frac{45}{8}}$

(b) $\int \frac{\cos(\sqrt{t})}{\sqrt{t}\sin^2(\sqrt{t})}\,dt = \int \frac{1}{u^2}\cdot 2\,du$

Hint: u-sub.

u-sub box

$u = \sin(\sqrt{t})$

$du = \frac{1}{2\sqrt{t}}\cos(\sqrt{t})\,dt$

$2du = \frac{\cos(\sqrt{t})}{\sqrt{t}}\,dt$

$= -\frac{2}{u} + C$

$= \boxed{\frac{-2}{\sin(\sqrt{t})} + C}$

GLC

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$u = \sin x$ $\quad (\cos^2 x)^2 = (1-\sin^2 x)^2$ swap to sin $\quad du$

(c) $\int \sin^2(x)\cos^5(x)\,dx = \int \sin^2 x \cdot \cos^4 x \cdot \cos x\,dx$

U-sub Box

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$$= \int u^2 (1-u^2)^2 \cdot du$$

$$= \int u^2(1-2u^2+u^4)\,du$$

$$= \int u^2 - 2u^4 + u^6\,du$$

$$= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$$

$$= \boxed{\frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C}$$

(d) $\int xe^{2x}\,dx$

IBP Box

$$\boxed{\begin{aligned} u &= x & dv &= e^{2x}\,dx \\ du &= dx & v &= \tfrac{1}{2}e^{2x} \end{aligned}}$$

$$= \frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx$$

$$= \boxed{\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C}$$

GCC

7. (10 points) Suppose $f(x) = 2x - 1$. Use a general Riemann Sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

to evaluate the definite integral of $f(x)$ on the interval $[1, 3]$
(a) Find an expression for $R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$ that does not involve sigma's.
(b) Find the exact value by taking the limit of the expression you found in part (a).

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3-1}{n} = 2/n$

$x_k = a + k \cdot \Delta x$

$x_k = 1 + \frac{2k}{n}$

Please box your answer for each part and clearly organize your work.

(a)

$$R_n = \sum_{k=1}^{n} f(x_k) \cdot \Delta x = \sum_{k=1}^{n} \left[2\left(1 + \frac{2k}{n}\right) - 1\right] \cdot \frac{2}{n}$$

$$= \sum_{k=1}^{n} \frac{4k}{n} \cdot \frac{2}{n} + \frac{2}{n} = \frac{8}{n^2} \sum_{k=1}^{n} k + \frac{2}{n} \sum_{k=1}^{n} 1$$

$$= \frac{8}{n^2} \frac{n(n+1)}{2} + \frac{2}{\cancel{n}} \cdot \cancel{n} = \boxed{4 \frac{n(n+1)}{n^2} + 2} \text{ (a)}$$

(b)

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} 4 \frac{n(n+1)}{n^2} + 2 = 4 \cdot 1 + 2 = \boxed{6}$$

GLC