Math 1552

Section 8.3:
Powers and Products of Trigonometric Functions

Week	Mon	Tues	Wed	Thurs	Fri
1	May 15 Introduction to Math 1552 Section 4.8: Anti-derivatives	May 16 Calculus review WS 4.8	May 17 Sections 5.1-5.2: Area under the curve	May 18 WS 5.1 WS 5.2-5.3	May 19 Section 5.3: The Definite Integral
2	May 22 Section 5.3: The Definite Integral cont. Section 5.4: The Fundamental Theorem of Calculus	May 23 WS 5.2-5.3 cont. WS 5.3	May 24 Section 5.4: The Fundamental Theorem of Calculus cont. *Welcome survey and syllabus quiz due!*	May 25 WS 5.3 cont. Quiz #1 (4.8, 5.1-5.3)	May 26 Section 5.5: Integration by Substitution
3	May 29 **NO CLASS** Memorial Day	May 30 WS 5.4 WS 5.5-5.6	May 31 Section 5.6: Area Between Curves	Jun 1 WS 5.5-5.6 cont. WS 5.6 Quiz #2 (5.4-5.6)	Jun 2 Section 8.2: Integration by Parts
4	Jun 5 Section 8.3: Powers of Trig Functions	Jun 6 WS 8.2 WS 8.3	Jun 7 Review for Test 1	Jun 8 **Test #1 (4.8, 5.1-5.6, 8.2-8.3)**	Jun 9 Section 8.4: Trigonometric Substitution

Review Question: Which integrals can we evaluate *by parts*?

u-sub

IBP

basic anti-deriv.

(A) $\int \frac{x^2}{1+x^3}dx$ $\quad u = 1+x^3$, $du = 3x^2\,dx$

(B) $\int \frac{1}{x}e^{\ln x}dx = \int \frac{1}{x}\cdot x\,dx = \int 1\,dx$

(C) $\int x^5 e^{x^3}dx = \int \underbrace{x^3}_{u}\cdot \underbrace{x^2 e^{x^3}\,dx}_{dv}$

(D) $\int \underbrace{x}_{dv}\,\underbrace{\tan^{-1}(x)}_{u}\,dx$

Exam 1 on Thursday

Practice Exam will be posted on personal website (w/ key!)

Quiz #2 posted (Section G)

Case 1 If m **is odd**, we write m as $2k+1$ and use the identity $\sin^2 x = 1-\cos^2 x$ to obtain

$$\sin^m x = \sin^{2k+1}x = (\sin^2 x)^k \sin x = (1-\cos^2 x)^k \sin x. \quad (1)$$

Then we combine the single $\sin x$ with dx in the integral and set $\sin x\,dx$ equal to $-d(\cos x)$.

Case 2 If n **is odd** in $\int \sin^m x \cos^n x\,dx$, we write n as $2k+1$ and use the identity $\cos^2 x = 1-\sin^2 x$ to obtain

$$\cos^n x = \cos^{2k+1}x = (\cos^2 x)^k \cos x = (1-\sin^2 x)^k \cos x.$$

We then combine the single $\cos x$ with dx and set $\cos x\,dx$ equal to $d(\sin x)$.

Case 3 If **both** m **and** n **are even** in $\int \sin^m x \cos^n x\,dx$, we substitute

$$\sin^2 x = \frac{1-\cos 2x}{2}, \qquad \cos^2 x = \frac{1+\cos 2x}{2} \quad (2)$$

to reduce the integrand to one in lower powers of $\cos 2x$.

Ex 1

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1-\cos^2 x$

$$\int \sin^3 x\cos^2 x\,dx. = \int \underbrace{\cos^2 x}_{u^2}\cdot \underbrace{\sin^2 x}_{??}\cdot \underbrace{\sin x\,dx}_{-du}$$

$$= \int \underbrace{\cos^2 x}_{u^2}\underbrace{(1-\cos^2 x)}_{(1-u^2)}\cdot \underbrace{\sin x\,dx}_{-du}$$

u-sub Box
$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$$= -\int u^2(1-u^2)\,du$$

???

$$= \int -u^2+u^4\,du = -\frac{1}{3}u^3+\frac{1}{5}u^5+C$$

$$= -\frac{1}{3}(\cos x)^3+\frac{1}{5}(\cos x)^5+C$$

$$= \boxed{-\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+C}$$

(*) $\sin^2 x+\cos^2 x = 1$
(*) $1+\tan^2 x = \sec^2 x$

(1) very important

(*) $\sin^2 x = \frac{1}{2}[1-\cos(2x)]$
(*) $\cos^2 x = \frac{1}{2}[1+\cos(2x)]$
(*) $\sin(2x) = 2\sin x\cos x$

(2) Somewhat important.

$\sin x\cos y = \frac{1}{2}[\sin(x-y)+\sin(x+y)]$

$\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$

$\cos x\cos y = \frac{1}{2}[\cos(x-y)+\cos(x+y)]$

Ex 2

$$\int \cos^5 x\,dx. = \int \cos^4 x\cdot\cos^1 x\,dx$$

u-sub
$u = \sin x$
$du = \cos x\,dx$

$$= \int (\cos^2 x)^2\cos x\,dx$$

$$= \int (1-\sin^2 x)^2\cos x\,dx = \int (1-u^2)^2\cdot du$$

$$\int 1-2u^2+u^4\,du = u-\frac{2}{3}u^3+\frac{1}{5}u^5+C$$

$$= \boxed{\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C}$$

Start w/ double-angle.

$$\int \sin^2 x \cos^4 x \, dx.$$

$(*) \sin^2 x + \cos^2 x = 1$

$(*) 1 + \tan^2 x = \sec^2 x$

$(*) \sin^2 x = \frac{1}{2}[1 - \cos(2x)]$

$(*) \cos^2 x = \frac{1}{2}[1 + \cos(2x)]$

$(*) \sin(2x) = 2 \sin x \cos x$

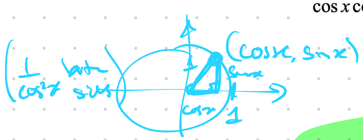
$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$$\sin^2 x + \cos^2 x = 1.$$

$$\Rightarrow \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$



$$\Rightarrow \boxed{\tan^2 x + 1 = \sec^2 x} \longrightarrow (*) \boxed{\tan^2 x = \sec^2 x - 1}$$

Ex 3

$$\int \tan^4 x \, dx.$$

$$= \int \tan^2 x \cdot \tan^2 x \, dx$$

$$\downarrow (*)$$

$$= \int \tan^2 x (\sec^2 x - 1) \, dx$$

$$= \int \tan^2 x \cdot \sec^2 x - \tan^2 x \, dx$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \tan^2 x \, dx$$

$$\downarrow (*)$$

$$= \int \tan^2 x \sec^2 x \, dx - \int \sec^2 x - 1 \, dx$$

$u = \tan x$
$du = \sec^2 x \, dx$

$$= \int u^2 \, du - (\tan x - x) + C$$

$$= \boxed{\frac{1}{3} \tan^3 x - \tan x + x + C}$$

$$\int \sec^3 x \, dx.$$

Start w/ double-angle.

$$\int \sin^2 x \cos^4 x \, dx.$$

(*) $\sin^2 x + \cos^2 x = 1$

(*) $1 + \tan^2 x = \sec^2 x$

(*) $\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$

(*) $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$

(*) $\sin(2x) = 2\sin x \cos x$

$\sin x \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

Ex 4

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$$\int \sec^3 x \, dx. = \int \sec x \cdot \sec^2 x \, dx$$

$$= \int \sec x (\tan^2 x + 1) \, dx$$

$$= \int \sec x \tan^2 x + \sec x \, dx$$

$$= \int \tan^2 x \sec x \, dx + \int \sec x \, dx$$

IBP wrap around

$$\int e^x \cos x \, dx$$

u-sub

$\frac{\sec x + \tan x}{\sec x + \tan x}$

$$= \underbrace{\int \tan x \cdot \sec x \tan x \, dx}_{\text{IBP?}} + \ln|\sec x + \tan x| + C$$

$\clubsuit = \square - \clubsuit + \square + C$

$u = \tan x \qquad dv = \sec x \tan x \, dx$

$du = \sec^2 x \, dx \qquad v = \sec x$

$$= \sec x \tan x - \int \sec x \cdot \sec^2 x \, dx + \ln|\sec x + \tan x| + C$$

$$\Rightarrow \int \sec^3 x \, dx = \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C}$$

$\tan^2 x + 1 = \sec^2 x \rightarrow \cancel{\sec^2 x = 1 - \tan^2 x}$

EX 5

$$\int \tan^4 x \sec^4 x \, dx. = \int \tan^4 x \left(\sec^2 x\right) \cdot \sec^2 x \, dx$$

u-sub

$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x \, dx \end{aligned}}$

$$= \int \tan^4 x \left(1 + \tan^2 x\right) \sec^2 x \, dx$$

$$= \int u^4 \left(1 + u^2\right) du$$

$$= \int u^4 + u^6 \, du$$

$$= \frac{1}{5} u^5 + \frac{1}{7} u^7 + C$$

$$= \boxed{\frac{1}{5} \tan^5 x + \frac{1}{7} \tan^7 x + C}$$

$$\int \cos^2(x) \cot(x) dx$$

$$\int \sin^4(x) dx$$

Ex.6

$\sin\left(3\cdot\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)$

$\sin(3\cdot 0)=\sin(0)=0$

$$\int_0^{\pi/6} 3\cos^5(3x)\,dx$$

Evaluate the integral.

$\int \sin^2(x)\cos^3(x)dx$

$(A)\ \frac{1}{5}\sin^5(x)+C$

$(B)\ \frac{1}{3}\sin^3(x)-\frac{1}{5}\sin^5(x)+C$

$(C)\ \frac{1}{12}\sin^3(x)\cos^4(x)+C$

$(D)\ -\frac{1}{3}\cos^3(x)+\frac{1}{5}\cos^5(x)+C$

$$=\int_0^{\pi/6}\underbrace{\left(\cos^2(3x)\right)^2}_{(1-\sin^2 3x)^2}\underbrace{3\cos(3x)\,dx}_{du}$$

u-sub

$u=\sin(3x)$

$du=3\cos(3x)\,dx$

$$=\int_*^*(1-u^2)^2\,du=\int 1-2u^2+u^4\,du$$

$$=u-\frac{2}{3}u^3+\frac{1}{5}u^5\Big|_*^* = \sin(3x)-\frac{2}{3}\sin^3(3x)+\frac{1}{5}\sin^5(3x)\Big|_0^{\pi/6}$$

$$=\left(1-\frac{2}{3}\cdot1^3+\frac{1}{5}1^5\right)-\left(0-0+0\right)$$

$$=\frac{1}{3}+\frac{1}{5}=\boxed{8/15}$$

$$\int \tan^3(x)\sec^3(x)dx$$

$\sin^2x+\cos^2x=1$

$\tan^2x+1=\sec^2x$

?? u-sub

$u=\tan x$

$du=\sec^2x\,dx$

!!

u-sub

$u=\sec x$

$du=\sec x\tan x\,dx$

$$=\int\underbrace{\tan^2x}_{\sec^2x-1}\overbrace{\sec^2x}^{u^2}\cdot\underbrace{\sec x\tan x\,dx}_{du}$$

$$=\int(\sec^2x-1)\sec^2x\cdot\sec x\tan x\,dx$$

$$=\int(u^2-1)\cdot u^2\cdot du$$

$$\int \tan^3(x)\sec^3(x)dx$$

?? u-sub

$u = \tan x$
$du = \sec^2 x \, dx$

u-sub !!

$u = \sec x$
$du = \sec x \tan x \, dx$

$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$

$$= \int \underbrace{\tan^2 x}_{\sec^2 x - 1} \overbrace{\sec^2 x}^{u^2} \cdot \underbrace{\sec x \tan x \, dx}_{du}$$

$$= \int (\sec^2 x - 1)\sec^2 x \cdot \sec x \tan x \, dx$$

$$= \int (u^2 - 1) \cdot u^2 \cdot du$$

$$= \int u^4 - u^2 \, du = \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

* FTC problems.
* Upper/lower/left/right RS

Ex. 6

$$\sin\left(3\cdot\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)$$

$$\sin(3\cdot 0)=\sin(0)=0$$

$$\int_0^{\pi/6} 3\cos^5(3x)\,dx$$

$$= \int_0^{\pi/6} \overbrace{\left(\cos^2(3x)\right)^2}^{(1-\sin^2 3x)^2} \underbrace{3\cos(3x)\,dx}_{du}$$

u-sub

$u=\sin(3x)$

$du=3\cos(3x)\,dx$

$$= \int_*^* (1-u^2)^2\,du = \int 1-2u^2+u^4\,du$$

$$= u-\frac{2}{3}u^3+\frac{1}{5}u^5\Big|_*^* = \sin(3x)-\frac{2}{3}\sin^3(3x)+\frac{1}{5}\sin^5(3x)\Big|_0^{\pi/6}$$

$$= \left(1-\frac{2}{3}\cdot 1^3+\frac{1}{5}1^5\right)-(0-0+0)$$

$$= \frac{1}{3}+\frac{1}{5} = \boxed{8/15}$$

Evaluate the integral.

$\int \sin^2(x)\cos^3(x)dx$

(A) $\frac{1}{5}\sin^5(x)+C$

(B) $\frac{1}{3}\sin^3(x)-\frac{1}{5}\sin^5(x)+C$

(C) $\frac{1}{12}\sin^3(x)\cos^4(x)+C$

(D) $-\frac{1}{3}\cos^3(x)+\frac{1}{5}\cos^5(x)+C$

$\int \tan^3(x)\sec^3(x)dx$

$\int \sec^3(x)dx$

$\int \tan^3(x)dx$

EXERCISES 8.3

Powers of Sines and Cosines

Evaluate the integrals in Exercises 1–22.

1. $\int \cos 2x\,dx$

2. $\int_0^{\pi} 3\sin\frac{x}{3}\,dx$

3. $\int \cos^3 x \sin x\,dx$

4. $\int \sin^4 2x \cos 2x\,dx$

5. $\int \sin^3 x\,dx$

6. $\int \cos^3 4x\,dx$

7. $\int \sin^5 x\,dx$

8. $\int_0^{\pi} \sin^5\frac{x}{2}\,dx$

9. $\int \cos^3 x\,dx$

$\boxed{\textbf{10. } \int_0^{\pi/6} 3\cos^5 3x\,dx}$??

11. $\int \sin^3 x \cos^3 x\,dx$

12. $\int \cos^3 2x \sin^5 2x\,dx$

13. $\int \cos^2 x\,dx$

14. $\int_0^{\pi/2} \sin^2 x\,dx$

15. $\int_0^{\pi/2} \sin^7 y\,dy$

16. $\int 7\cos^7 t\,dt$

17. $\int_0^{\pi} 8\sin^4 x\,dx$

18. $\int 8\cos^4 2\pi x\,dx$

19. $\int 16\sin^2 x \cos^2 x\,dx$

20. $\int_0^{\pi} 8\sin^4 y \cos^2 y\,dy$

21. $\int 8\cos^3 2\theta \sin 2\theta\,d\theta$

22. $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta\,d\theta$

Integrating Square Roots

Evaluate the integrals in Exercises 23–32.

23. $\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\,dx$

24. $\int_0^{\pi} \sqrt{1-\cos 2x}\,dx$

25. $\int_0^{\pi} \sqrt{1-\sin^2 t}\,dt$

26. $\int_0^{\pi} \sqrt{1-\cos^2\theta}\,d\theta$

27. $\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{\sqrt{1-\cos x}}\,dx$

28. $\int_0^{\pi/6} \sqrt{1+\sin x}\,dx$

$\left(\textit{Hint:} \text{ Multiply by } \sqrt{\frac{1-\sin x}{1-\sin x}}.\right)$

29. $\int_{5\pi/6}^{\pi} \frac{\cos^4 x}{\sqrt{1-\sin x}}\,dx$

30. $\int_{\pi/2}^{3\pi/4} \sqrt{1-\sin 2x}\,dx$

31. $\int_0^{\pi/2} \theta\sqrt{1-\cos 2\theta}\,d\theta$

32. $\int_{-\pi}^{\pi} (1-\cos^2 t)^{3/2}\,dt$

Powers of Tangents and Secants

Evaluate the integrals in Exercises 33–50.

33. $\int \sec^2 x \tan x\,dx$

34. $\int \sec x \tan^2 x\,dx$

35. $\int \sec^3 x \tan x\,dx$

36. $\int \sec^3 x \tan^3 x\,dx$

37. $\int \sec^2 x \tan^2 x\,dx$

38. $\int \sec^4 x \tan^2 x\,dx$

39. $\int_{-\pi/3}^{0} 2\sec^3 x\,dx$

40. $\int e^x \sec^3 e^x\,dx$

FTC

$$\frac{d}{dx}\left(\underbrace{\int_{2x}^{\sqrt{x}} t^4 + \frac{3}{\sqrt{1-t^2}}\, dt}_{F(x)}\right)$$

$$G(x) = \int_0^x t^4 + \frac{3}{\sqrt{1-t^2}}\, dt$$

$$G'(x) = x^4 + \frac{3}{\sqrt{1-x^2}}$$

by FTC

$$F(x) = G(\sqrt{x}) - G(2x)$$

$$\Rightarrow F'(x) = G'(\sqrt{x}) \cdot (\sqrt{x})' - G'(2x) \cdot (2x)'$$

$$= \left((\sqrt{x})^4 + \frac{3}{\sqrt{1-(\sqrt{x})^2}}\right) \frac{1}{2\sqrt{x}} - \left((2x)^4 + \frac{3}{\sqrt{1-(2x)^2}}\right) 2$$

$$= \left(x^2 + \frac{3}{\sqrt{1-x}}\right) \frac{1}{2\sqrt{x}} - 32x^4 - \frac{6}{\sqrt{1-4x^2}}$$

FTC

$$F(x) = \int_0^{\sqrt{x}} \boxed{t^4} + \boxed{\frac{3}{\sqrt{1-t^2}}} \, dt$$

$$\Rightarrow F(x) = \frac{1}{5}t^5 + 3\sin^{-1}(t) \Big|_0^{\sqrt{x}}$$

$$\Rightarrow F(x) = \frac{1}{5}(\sqrt{x})^5 + 3\sin^{-1}(\sqrt{x})$$

$$\Rightarrow F'(x) = \boxed{(\sqrt{x})^4} \cdot (\sqrt{x})' + \boxed{\frac{3}{\sqrt{1-(\sqrt{x})^2}}} (\sqrt{x})'$$

EXERCISES 5.4

Evaluating Integrals

Evaluate the integrals in Exercises 1–34.

1. $\int_0^2 x(x-3)\,dx$

2. $\int_{-1}^1 (x^2-2x+3)\,dx$

3. $\int_{-2}^2 \frac{3}{(x+3)^4}\,dx$

4. $\int_{-1}^1 x^{299}\,dx$

5. $\int_1^4 \left(3x^2-\frac{x^3}{4}\right)dx$

6. $\int_{-2}^3 (x^3-2x+3)\,dx$

7. $\int_0^1 (x^2+\sqrt{x})\,dx$

8. $\int_1^{32} x^{-6/5}\,dx$

9. $\int_0^{\pi/3} 2\sec^2 x\,dx$

10. $\int_0^{\pi} (1+\cos x)\,dx$

11. $\int_{\pi/4}^{3\pi/4} \csc\theta\cot\theta\,d\theta$

12. $\int_0^{\pi/3} 4\frac{\sin u}{\cos^2 u}\,du$

13. $\int_{\pi/2}^0 \frac{1+\cos 2t}{2}\,dt$

14. $\int_{-\pi/3}^{\pi/3} \sin^2 t\,dt$

15. $\int_0^{\pi/4} \tan^2 x\,dx$

16. $\int_0^{\pi/6} (\sec x+\tan x)^2\,dx$

17. $\int_0^{\pi/8} \sin 2x\,dx$

18. $\int_{-\pi/3}^{-\pi/4}\left(4\sec^2 t+\frac{\pi}{t^2}\right)dt$

19. $\int_1^{-1} (r+1)^2\,dr$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)\,dt$

21. $\int_{\sqrt{2}}^1 \left(\frac{u^7}{2}-\frac{1}{u^5}\right)du$

22. $\int_{-3}^{-1} \frac{y^5-2y}{y^3}\,dy$

23. $\int_1^{\sqrt{2}} \frac{s^2+\sqrt{s}}{s^2}\,ds$

24. $\int_1^8 \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}}\,dx$

25. $\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x}\,dx$

26. $\int_0^{\pi/3} (\cos x+\sec x)^2\,dx$

27. $\int_{-4}^4 |x|\,dx$

28. $\int_0^{\pi} \frac{1}{2}(\cos x+|\cos x|)\,dx$

29. $\int_0^{\ln 2} e^{3x}\,dx$

30. $\int_1^2 \left(\frac{1}{x}-e^{-x}\right)dx$

31. $\int_0^{1/2} \frac{4}{\sqrt{1-x^2}}\,dx$

32. $\int_0^{1/\sqrt{3}} \frac{dx}{1+4x^2}$

33. $\int_2^4 x^{\pi-1}\,dx$

34. $\int_{-1}^0 \pi^{x-1}\,dx$

In Exercises 35–38, guess an antiderivative for the integrand function. Validate your guess by differentiation and then evaluate the given definite integral. (*Hint:* Keep the Chain Rule in mind when trying to guess an antiderivative. You will learn how to find such antiderivatives in the next section.)

35. $\int_0^1 xe^{x^2}\,dx$

36. $\int_1^2 \frac{\ln x}{x}\,dx$

37. $\int_2^5 \frac{x\,dx}{\sqrt{1+x^2}}$

38. $\int_0^{\pi/3} \sin^2 x\cos x\,dx$

Derivatives of Integrals

Find the derivatives in Exercises 39–44.

a. by evaluating the integral and differentiating the result.

b. by differentiating the integral directly.

39. $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\,dt$

40. $\frac{d}{dx}\int_1^{\sin x} 3t^2\,dt$

41. $\frac{d}{dt}\int_0^{t^4} \sqrt{u}\,du$

42. $\frac{d}{d\theta}\int_0^{\tan\theta} \sec^2 y\,dy$

43. $\frac{d}{dx}\int_0^{x^3} e^{-t}\,dt$

44. $\frac{d}{dt}\int_0^{\sqrt{t}}\left(x^4+\frac{3}{\sqrt{1-x^2}}\right)dx$

Find dy/dx in Exercises 45–56.

45. $y=\int_0^x \sqrt{1+t^2}\,dt$

46. $y=\int_1^x \frac{1}{t}\,dt,\quad x>0$

47. $y=\int_{\sqrt{x}}^0 \sin(t^2)\,dt$

48. $y=x\int_2^{x^2} \sin(t^3)\,dt$

49. $y=\int_{-1}^x \frac{t^2}{t^2+4}\,dt-\int_3^x \frac{t^2}{t^2+4}\,dt$

50. $y=\left(\int_0^x (t^3+1)^{10}\,dt\right)^3$

51. $y=\int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}},\quad |x|<\frac{\pi}{2}$

52. $y=\int_{\tan x}^0 \frac{dt}{1+t^2}$

53. $y=\int_0^{e^{x^2}} \frac{1}{\sqrt{t}}\,dt$

54. $y=\int_{2^x}^1 \sqrt[3]{t}\,dt$

55. $y=\int_0^{\sin^{-1}x} \cos t\,dt$

56. $y=\int_{-1}^{x^{1/\pi}} \sin^{-1} t\,dt$

Area

In Exercises 57–60, find the total area between the region and the x-axis.

57. $y=-x^2-2x,\quad -3\le x\le 2$

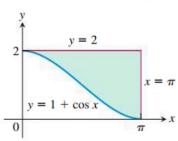
58. $y=3x^2-3,\quad -2\le x\le 2$

59. $y=x^3-3x^2+2x,\quad 0\le x\le 2$

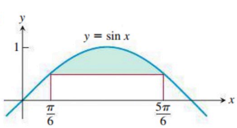
60. $y=x^{1/3}-x,\quad -1\le x\le 8$

Find the areas of the shaded regions in Exercises 61–64.

61.



62.



$\int \tan^2 x\,dx$

$= \int \frac{\sin^2 x}{\cos^2 x}\,dx$

$= \int \frac{1-\cos^2 x}{\cos^2 x}\,dx$

$= \int \sec^2 x - 1\,dx$

* wrap around IBP problem 80%

* area between curves 50%
  ↳ three interval problem 50%

* General Riemann sum 15%

Ex. (wrap around)

trig exp.

ILATE

Integrate using IBP

IBP Formula

$\int u\,dv = uv - \int v\,du$

$$I = \int e^{2x} \sin(3x)\,dx$$

IBP Box #1

$u = \sin(3x)$	$dv = e^{2x}\,dx$
$du = 3\cos(3x)\,dx$	$v = \frac{1}{2}e^{2x}$

$$= \sin(3x)\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\cdot 3\cos(3x)\,dx$$

$$= \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{2}\int e^{2x}\cos(3x)\,dx$$

IBP BOX #2

$u = \cos(3x)$	$dv = e^{2x}\,dx$
$du = -3\sin(3x)$	$v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{2}\left[\frac{1}{2}e^{2x}\cos(3x) - \int \frac{1}{2}e^{2x}\cdot(-3\sin(3x))\,dx\right]$$

$$= \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x) - \frac{9}{4}\int e^{2x}\sin(3x)\,dx$$

IDEA

$I = \text{stuff} - \frac{9}{4}I$

$I + \frac{9}{4}I = \text{stuff}$

$$I + \frac{9}{4}I = \frac{4}{4}I + \frac{9}{4}I = \frac{13}{4}I$$

$$\int e^{2x}\sin 3x\,dx + \frac{9}{4}\int e^{2x}\sin 3x\,dx = \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x)$$

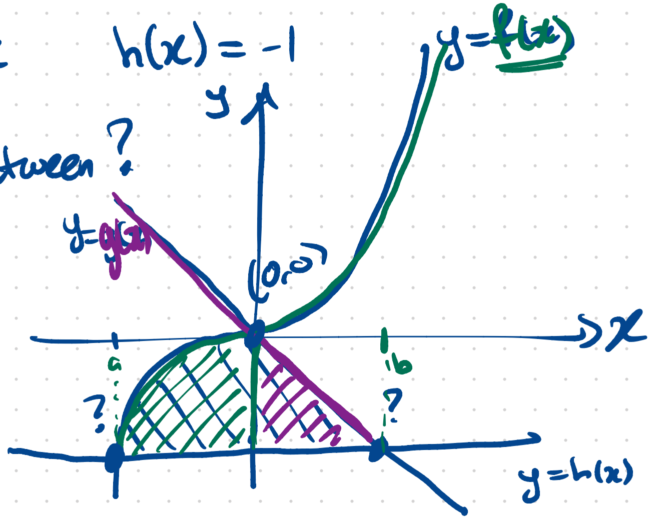
$\frac{4}{13}\cdot\frac{13}{4}$

$$\boxed{I = \frac{4}{13}\left(\frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x)\right) + C}$$

Studio worksheet 5.6

$f(x) = x^3 \qquad g(x) = -x \qquad h(x) = -1$

Q: Find the area between?



Green area

$$\int_a^0 f(x) - h(x)\, dx$$

purple area

$$\int_0^b g(x) - h(x)\, dx$$

to find a: Set y=y

$f(x) = h(x)$ & solve for x

$x^3 = -1 \Rightarrow x = -1$ So, $\displaystyle\int_{-1}^{0} x^3 - (-1)\, dx = \int_{-1}^{0} x^3 + 1\, dx$

$= \frac{1}{4}x^4 + x \Big|_{-1}^{0}$

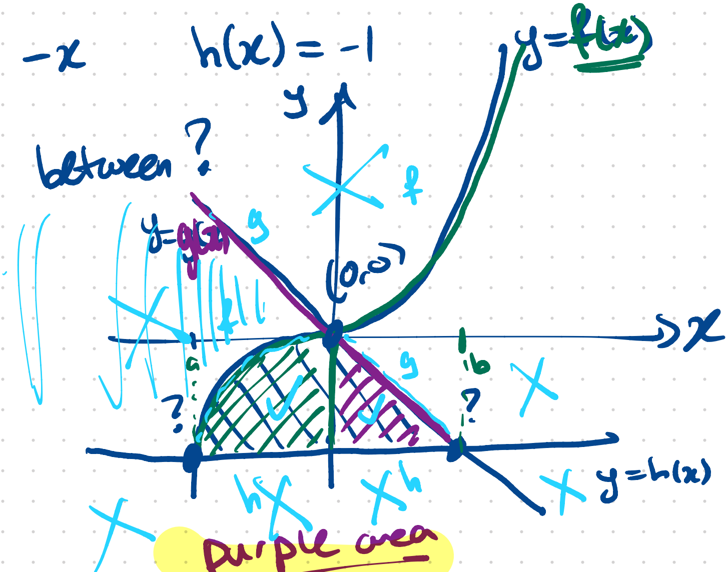
$= \left(\frac{1}{4}0^4 + 0\right) - \left(\frac{1}{4}(-1)^4 + (-1)\right)$

$= 0 - \left(\frac{1}{4} - 1\right) = \boxed{\frac{3}{4}}$

Studio worksheet 5.6

$f(x) = x^3 \qquad g(x) = -x \qquad h(x) = -1$

Q: Find the area between?



$\boxed{\text{area between } f \text{ \& } g \quad \int_a^b \text{TOP} - \text{BOT}\, dx}$

Green area

$$\int_a^0 f(x) - h(x)\, dx$$

purple area

$$\int_0^b g(x) - h(x)\, dx$$

to find b set y=y

$g(x) = h(x)$

$-x = -1 \Rightarrow x = 1$

So, $\int_0^1 (-x) - (-1)\, dx = \int_0^1 -x + 1\, dx$

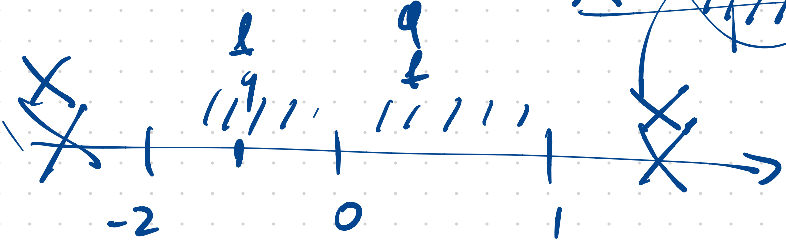
$= -\frac{1}{2}x^2 + x \Big|_0^1 = \left(-\frac{1}{2} + 1\right) = \boxed{\frac{1}{2}}$

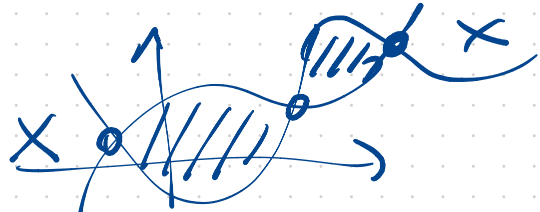
total area

total area ANS

$$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \boxed{\frac{5}{4}}$$

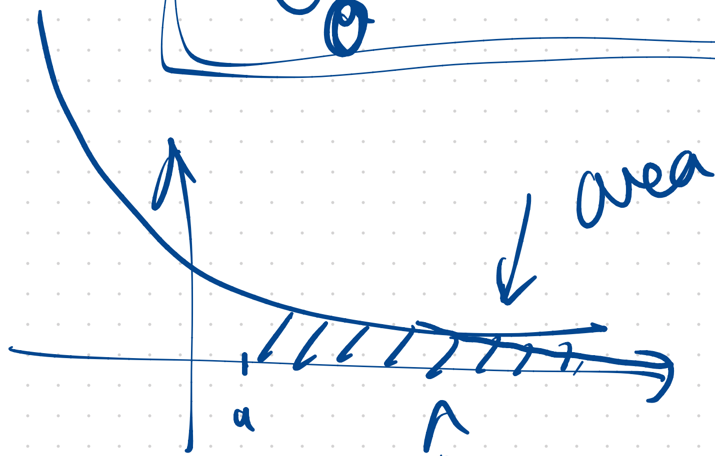
f g

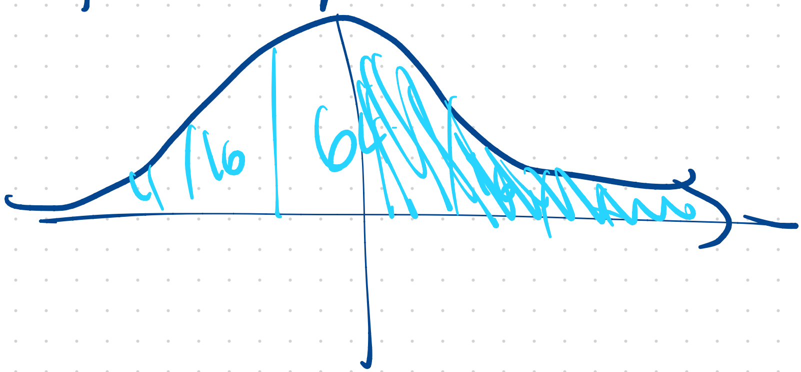




Normal dist.

$$\boxed{\int_0^\infty \boxed{e^{-x^2/\sigma^2}}\, dx} = 50\%$$





1. (2 points) Suppose that $f(x)$ is a function which is non-negative on the interval $[3, 5]$ and $\int_3^5 f(x)\, dx = A$. Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
○	○	If we estimate A using the left-endpoint method with $n = 4$ rectangles, then we will always get a value smaller than A.
○	○	If we estimate A using the upper-sum method with $n = 4$ rectangles, then we will always get a value at least as large as A.

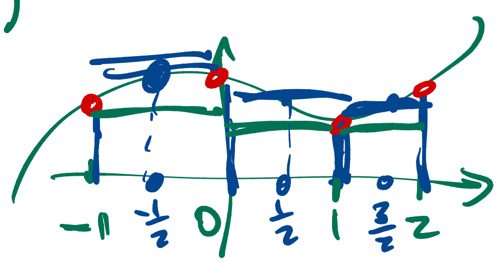
2. (3 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-2}^{2} f(x)\, dx = 10$, $\int_0^3 g(x)\, dx = 4$, and $\int_2^3 g(x)\, dx = 1$, find $A = \int_0^2 2f(x) - 3g(x)\, dx$.

$A = \boxed{}$

midpoint

plug in $\frac{x_i + x_{i+1}}{2}$

the midpoint x-value for each rectangle.



3. (3 points) Find the **lower-sum** Riemann sum estimate for the area between the x-axis and the function $f(x) = -x^2 + x + 3$ over the interval $[-1, 2]$ using $n = 3$ rectangles. area $\approx \boxed{5}$

given
$a = -1$
$b = 2$
$n = 3$

find Δx

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1$

$\Delta x = 1$ $\quad x_0 = -1 \quad x_1 = 0 \quad x_2 = 1 \quad x_3 = 2$

find x-values Δx

plug in x-values to get y-values.

$f(-1) = -(-1)^2 + (-1) + 3 = 1$
$f(0) = 3$
$f(1) = -(1)^2 + 1 + 3 = 3$
$f(2) = -(2)^2 + 2 + 3 = 1$

1
3
1

LAST Step. add up y-values
Mult. by Δx's.

Δx

$1(1) + 3(1) + 1(1) = \boxed{5}$

(c) $\int \tan^3(x)\sec^3(x)\,dx$

Very Hard

(d) $I = \int x^3\sqrt{x^2+1}\,dx = \int \underbrace{x^2}_{u} \cdot \underbrace{x\sqrt{x^2+1}\,dx}_{dv}$

Hint: integration by parts.

x^3

$u = x^2 \qquad dv = x\sqrt{x^2+1}\,dx$

$du = 2x\,dx \qquad v = \frac{1}{3}(x^2+1)^{3/2}$

d/dx

integrate

$\int u\,dv = uv - \int v\,du$

IBP

$\int \overbrace{x\sqrt{x^2+1}\,dx}^{dv}$

$= \int u^{1/2}\cdot\frac{1}{2}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}u^{3/2} + C = \underbrace{\frac{1}{3}(x^2+1)^{3/2}}_{v} + C$

u-sub

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$I = x^2\cdot\frac{1}{3}(x^2+1)^{3/2} - \int \frac{1}{3}(x^2+1)^{3/2}\cdot 2x\,dx$

$= \frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{3}\int x\cdot(x^2+1)^{3/2}\,dx$

same box.

$u = x^2+1$

$= \frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{3}\int u^{3/2}\cdot\frac{1}{2}\,du$

$= \frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{3}\cdot\frac{1}{2}\cdot\frac{2}{5}u^{5/2} + C$

$= \boxed{\frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + C}$

Sample Exam.

~~**Math 1552**~~
Summer 2022
Test 1
~~**June 9, 2022**~~
Time Limit: 50 Minutes

Name (Print): ____________________

Canvas email: ____________________

Teaching Assistant/Section: ____________________

GT ID:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: ____________________

1. (5 points) Let $f(x)$ be a function that is always increasing in the interval $[1, 5]$. Suppose $\int_1^5 f(x)dx = 10$.
If we use 4 sub-intervals and left-hand endpoints to estimate the area under the curve $f(x)$ bounded by $x = 1$ and $x = 5$, would it be larger or smaller than 10? Explain your answer.

2. (5 points) Let $f(x)$ be an even function. Suppose that $\int_{-5}^{5} f(x)dx = 20$ and $\int_8^5 f(x)dx = -2$. Find $\int_0^8 f(x)dx$?

b/c f even

$$\int_0^5 f(x)\,dx = 10$$

$$\int_5^8 f(x)\,dx = 2$$

prop of def. int.

prop of def. int.

$$\int_0^8 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^8 f(x)\,dx$$

$$= 10 + 2 = \boxed{12}$$