WEEK 11

Volumes of Revolution

Disks/Washers & Cylindrical Shells

Math 1552

Sections 6.1-6.2:

Volumes of Revolution

8	Jul 3 **NO CLASS** Independence Day	Jul 4 **NO CLASS** Student Recess	Jul 5 Section 10.6: cont. Section 10.7: Power series	Jul 6 WS 10.4 WS 10.5 Quiz #5 (10.4-10.5)	Jul 7 Section 10.7, cont.
9	Jul 10 Sections 10.8-10.9: Taylor polynomials and series	Jul 11 WS 10.6 WS 10.7	Jul 12 Sections 10.8-10.9, cont.	Jul 13 WS 10.8-10.9 Quiz #6 (10.6-10.8)	Jul 14 Sections 10.8-10.9, cont.
10	Jul 17 Sections 10.8-10.9, cont.	Jul 18 WS 10.8-10.9 (3 versions)	Jul 19 Sections 10.8-10.9, cont.	Jul 20 **Test #3 (10.4-10.9)**	Jul 21 Section 6.1: Volumes by Disks
11	Jul 24 Section 6.1: Volumes by Cylindrical Shells Final Review	Jul 25 WS 6.1-6.2 *Last day for MML homework*	Jul 26 Reading Day	Jul 27	Jul 28 **FINAL EXAM** **11:20 AM – 2:10 PM**
12	Jul 31	Aug 1	Aug 2	Aug 3	Aug 4

Learning Goals

- Set up and evaluate integrals using the disk method
- Set up and evaluate integrals using the method of cylindrical shells
- Apply the "washer" method to either method above
- Adjust the standard formulas to rotate a region around any horizontal or vertical line

Example 1:

Find the volume of the solid generated by revolving the region bounded by $y=5x$, $x=0$, and $y=5$ about the y-axis.

Important Notes about Disks:

- The variable of integration *always* matches the axis of revolution.
- If you revolve about a line other than the x- or y-axis, you will need to adjust the formula to find the new radius.
- If you revolve a region bounded by two curves, you will need to apply the *washer method*.

CLOs
* Canvas
* Piazza
* Lecture
* Tell TAs to tell Studio
* Lecture Notes
* T-shirt

Ex. Find the volume of the solid obtained by revolving the shape below about the x-axis (Section 6.1 #17)

Example 2:

Find the volume of the solid generated by revolving the region bounded by

$y = \sqrt{1 - x^2}$ and $x + y = 1$

about the x-axis.

Example 3:

Find the volume of the solid generated by revolving the region bounded by:

$y = \sqrt{x+1}, x = 3,$ and the x-axis about the line $y = -1.$

Example: Set up the integral to find the volume bounded by

$y = x + 2$ and $y = x^2, x \geq 0,$

about the x-axis.

(A) $V = \pi \int_{-1}^{2} \left[(x+2)^2 - (x^2)^2 \right] dx$

(B) $V = \pi \int_{0}^{2} \left[(x+2)^2 - (x^2)^2 \right] dx$

(C) $V = \pi \int_{-1}^{2} \left[(x^2)^2 - (x+2)^2 \right] dx$

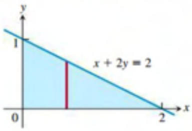
(D) $V = \pi \int_{0}^{2} \left[(x^2)^2 - (x+2)^2 \right] dx$

Section 6.1: 9, 17, 21, 23, 29, 31, 43, 47, 53, 55

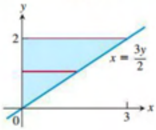
Volumes by the Disk Method

In Exercises 17–20, find the volume of the solid generated by revolving the shaded region about the given axis.

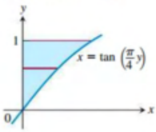
17. About the x-axis



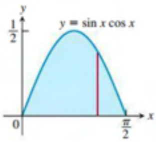
18. About the y-axis



19. About the y-axis



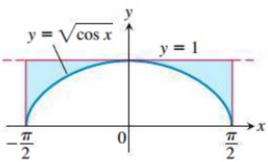
20. About the x-axis



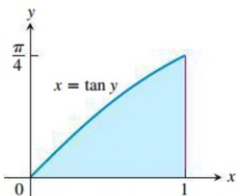
Volumes by the Washer Method

Find the volumes of the solids generated by revolving the shaded regions in Exercises 39 and 40 about the indicated axes.

39. The x-axis



40. The y-axis



Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 41–46 about the x-axis.

41. $y = x, \quad y = 1, \quad x = 0$

42. $y = 2\sqrt{x}, \quad y = 2, \quad x = 0$

43. $y = x^2 + 1, \quad y = x + 3$

44. $y = 4 - x^2, \quad y = 2 - x$

45. $y = \sec x, \quad y = \sqrt{2}, \quad -\pi/4 \le x \le \pi/4$

46. $y = \sec x, \quad y = \tan x, \quad x = 0, \quad x = 1$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 21–30 about the x-axis.

21. $y = x^2, \quad y = 0, \quad x = 2$

22. $y = x^3, \quad y = 0, \quad x = 2$

23. $y = \sqrt{9 - x^2}, \quad y = 0$

24. $y = x - x^2, \quad y = 0$

25. $y = \sqrt{\cos x}, \quad 0 \le x \le \pi/2, \quad y = 0, \quad x = 0$

26. $y = \sec x, \quad y = 0, \quad x = -\pi/4, \quad x = \pi/4$

27. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1$

28. The region between the curve $y = \sqrt{\cot x}$ and the x-axis from $x = \pi/6$ to $x = \pi/2$

29. The region between the curve $y = 1/(2\sqrt{x})$ and the x-axis from $x = 1/4$ to $x = 4$

30. $y = e^{x-1}, \quad y = 0, \quad x = 1, \quad x = 3$

In Exercises 31 and 32, find the volume of the solid generated by revolving the region about the given line.

31. The region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec x \tan x$, and on the left by the y-axis, about the line $y = \sqrt{2}$

32. The region in the first quadrant bounded above by the line $y = 2$, below by the curve $y = 2 \sin x$, $0 \le x \le \pi/2$, and on the left by the y-axis, about the line $y = 2$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 33–38 about the y-axis.

33. The region enclosed by $x = \sqrt{5}y^2, \quad x = 0, \quad y = -1, \quad y = 1$

34. The region enclosed by $x = y^{3/2}, \quad x = 0, \quad y = 2$

35. The region enclosed by $x = \sqrt{2 \sin 2y}, \quad 0 \le y \le \pi/2, \quad x = 0$

36. The region enclosed by $x = \sqrt{\cos(\pi y/4)}, \quad -2 \le y \le 0, \quad x = 0$

37. $x = 2/\sqrt{y + 1}, \quad x = 0, \quad y = 0, \quad y = 3$

38. $x = \sqrt{2y}/(y^2 + 1), \quad x = 0, \quad y = 1$

In Exercises 47–50, find the volume of the solid generated by revolving each region about the y-axis.

47. The region enclosed by the triangle with vertices (1, 0), (2, 1), and (1, 1)

48. The region enclosed by the triangle with vertices (0, 1), (1, 0), and (1, 1)

49. The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$

50. The region in the first quadrant bounded on the left by the circle $x^2 + y^2 = 3$, on the right by the line $x = \sqrt{3}$, and above by the line $y = \sqrt{3}$

In Exercises 51 and 52, find the volume of the solid generated by revolving each region about the given axis.

51. The region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, about the line $x = -1$

52. The region in the second quadrant bounded above by the curve $y = -x^3$, below by the x-axis, and on the left by the line $x = -1$, about the line $x = -2$

Volumes of Solids of Revolution

53. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about

a. the x-axis. **b.** the y-axis.

c. the line $y = 2$. **d.** the line $x = 4$.

54. Find the volume of the solid generated by revolving the triangular region bounded by the lines $y = 2x$, $y = 0$, and $x = 1$ about

a. the line $x = 1$. **b.** the line $x = 2$.

55. Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about

a. the line $y = 1$. **b.** the line $y = 2$.

c. the line $y = -1$.

56. By integration, find the volume of the solid generated by revolving the triangular region with vertices (0, 0), $(b, 0)$, $(0, h)$ about

a. the x-axis. **b.** the y-axis.

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Notes about Shell Method:

- In the shell method, the variable of integration is the *opposite* of the axis of revolution.
- To use the washer method with shells:

$$V = 2\pi\int_a^b x[f(x)-g(x)]dx = 2\pi\int_a^b x[top - bottom]dx$$

OR

$$V = 2\pi\int_a^b y[f(y)-g(y)]dy = 2\pi\int_a^b y[right - left]dy$$

Example 4:

Find the volume of the solid generated by revolving the region bounded by the curves:

$y = \sin x$, the x-axis, and the lines

$x = 0,\ x = \dfrac{\pi}{2}$ about the y-axis.

Example: Set up the integral to find the volume bounded by

$y = x + 2$ and $y = x^2$

about the line $x = 2$.

(A) $V = 2\pi \int_1^4 (y+2)\left[(y-2) - \sqrt{y}\right] dy$

(B) $V = 2\pi \int_{-1}^2 (x-2)\left[(x+2) - x^2\right] dx$

(C) $V = 2\pi \int_1^4 (2-y)\left[(y-2) - \sqrt{y}\right] dy$

(D) $V = 2\pi \int_{-1}^2 (2-x)\left[(x+2) - x^2\right] dx$