**Math 1552**
**Summer 2023**
**Quiz 1** ***QUP only***
**May 25**
**Due date: Sunday at 11:59PM**

**Name (Print):** Key

**Canvas email:**

Teaching Assistant/Section:

**GT ID:** | | | | | | | | |

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, texbook/course documents, websites, solving tools, or each other (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific.** List the name of anyone who helped you. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (5 points) Give the **general** anti-derivative of the following function:

$$f(x) = \frac{\sqrt{x}+1}{3x} + 2e^{2x} - \frac{1}{4+x^2}$$

$$f(x) = \frac{\sqrt{x}}{3x} + \frac{1}{3x} + 2e^{2x} - \frac{1}{4(1+(\frac{x}{2})^2)} = \frac{1}{3\sqrt{x}} + \frac{1}{3}\cdot\frac{1}{x} + 2e^{2x} - \frac{1}{4}\frac{1}{1+(\frac{x}{2})^2}$$

$$F(x) = \boxed{\frac{2}{3}\sqrt{x} + \frac{1}{3}\ln|x| + e^{2x} - \frac{1}{2}\cdot\tan^{-1}(x/2) + C}$$

2. (5 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-3}^{0} f(x)\,dx = 4$ and $\int_{0}^{3} g(x)\,dx = 5$, find $\int_{-3}^{3} f(x) + 2g(x)\,dx$.

$$\int_{-3}^{3} f(x) + 2g(x)\,dx = \underbrace{\int_{-3}^{3} f(x)\,dx}_{f \text{ even}} + 2\underbrace{\int_{-3}^{3} g(x)\,dx}_{g \text{ odd}}$$

$$= 2\int_{-3}^{0} f(x)\,dx + 0 = 2(4) = \boxed{8}$$

3. (10 points) Suppose $f(x) = x^2 - 1$. Use a general Riemann Sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

to evaluate the definite integral of $f(x)$ on the interval $[1, 2]$, by following these steps:

**(a)** Find the length of each subinterval $\Delta x$ in terms of $n$.

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = 1/n$$

$$\boxed{1/n}$$

**(b)** Evaluate $x_k^*$ as the right-hand endpoint of the subinterval.

$$x_k = a + k \cdot \Delta x = 1 + \frac{k}{n}$$

$$\boxed{1 + \frac{k}{n}}$$

**(c)** Evaluate the function at $x_k^*$, i.e. find $f(x_k^*)$. *Simplify.*

$$\boxed{\frac{k^2}{n^2} + \frac{2k}{n}}$$

$$f(x_k) = \left(1 + \frac{k}{n}\right)^2 - 1 = \frac{k^2}{n^2} + \frac{2k}{n} + 1 - 1$$

**(d)** Using the following summation formulas to simplify the sigma notation, find an expression for $R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$ that does not involve sigma's.

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad$ $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left(\frac{k^2}{n^2} + \frac{2k}{n}\right)\frac{1}{n} = \sum_{k=1}^{n} \frac{k^2}{n^3} + \frac{2k}{n^2}$$

$$= \frac{1}{n^3}\sum_{k=1}^{n} k^2 + \frac{2}{n^2} \cdot \sum_{k=1}^{n} k$$

$$= \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{\cancel{2}}{n^2} \frac{n(n+1)}{\cancel{2}} = \boxed{\frac{1}{6} \frac{n(n+1)(2n+1)}{n^3} + \frac{n(n+1)}{n^2}}$$

**(e)** Using the sum you found in the previous step, find the definite integral.

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{1}{6} \frac{n(n+1)(2n+1)}{n^3} + \frac{n(n+1)}{n^2}$$

$$= \frac{1}{6} \cdot 2 + 1 = \frac{1}{3} + 1 = \boxed{4/3}$$

Check w/ FTC

$$\int_1^2 x^2 - 1 \, dx$$

$$= \frac{1}{3}x^3 - x \Big|_1^2$$

$$= \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{7}{3} - 1 = 4/3$$